Chapter Two

Budgetary and Other Constraints
on Choice

Consumption Choice Sets

- A consumption choice set is the collection of all consumption choices available to the consumer.

- What constrains consumption choice?
  - Budgetary, time and other resource limitations.

Budget Constraints

- A consumption bundle containing x_1 units of commodity 1, x_2 units of commodity 2 and so on up to x_n units of commodity n is denoted by the vector $(x_1, x_2, \ldots, x_n)$.

- Commodity prices are $p_1, p_2, \ldots, p_n$.

Budget Constraints

- Q: When is a bundle $(x_1, \ldots, x_n)$ affordable at prices $p_1, \ldots, p_n$?

- A: When

$$p_1 x_1 + \ldots + p_n x_n \leq m$$

- where m is the consumer's (disposable) income.

Budget Constraints

- The bundles that are only *just affordable* form the consumer's budget constraint. This is the set

$$\{ (x_1,\ldots,x_n) \mid x_1 \geq 0, \ldots, x_n \geq 0 \text{ and }$$
$$p_1 x_1 + \ldots + p_n x_n = m \}.$$

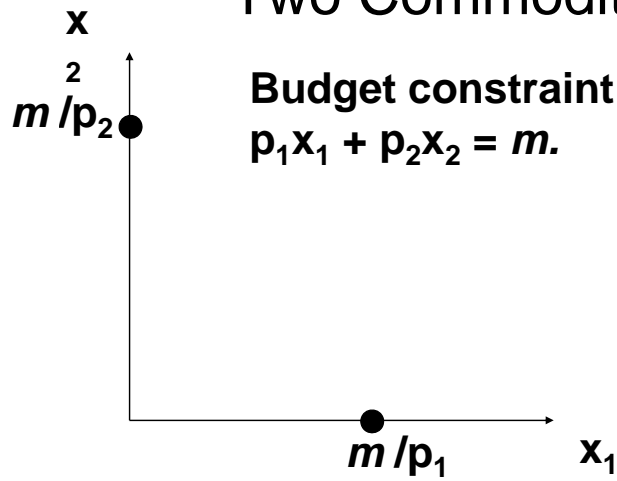- "Just Affordable" Note, equal sign (binding) and non-negative amounts

Budget Constraints

- The consumer's budget set is the set of all *affordable* bundles:

- $B(p_1, \ldots, p_n, m) = \{(x_1, \ldots, x_n) \mid x_1 \geq 0, \ldots, x_n \geq 0 \text{ and } p_1 x_1 + \ldots + p_n x_n \leq m \}$

- The budget constraint is the upper boundary of the budget set.
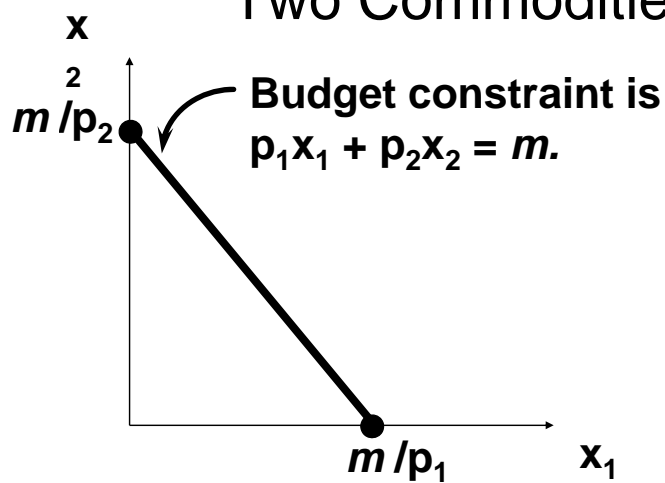- Let us work with that on the board!
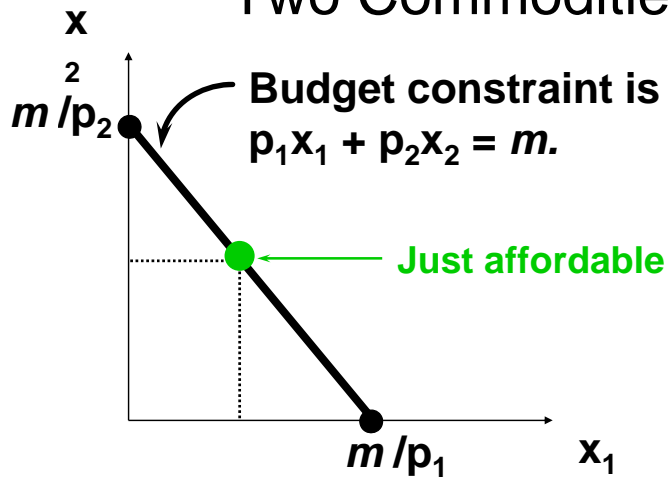
Budget Set and Constraint for Two Commodities

x_2

m/p_2 •

Budget constraint is
$p_1x_1 + p_2x_2 = m.$

• m/p_1 x_1

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Budget Set and Constraint for Two Commodities

x_2

m/p_2 •

Budget constraint is
$p_1x_1 + p_2x_2 = m.$

• m/p_1 x_1

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Budget Set and Constraint for Two Commodities

x_2

m/p_2

Budget constraint is
$p_1x_1 + p_2x_2 = m.$

Just affordable

m/p_1

x_1

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Budget Set and Constraint for Two Commodities

x_2

m/p_2

Budget constraint is
$p_1x_1 + p_2x_2 = m.$

Not affordable

Just affordable

m/p_1

x_1

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Budget Set and Constraint for Two Commodities

x_2

m/p_2

Budget constraint is
$p_1x_1 + p_2x_2 = m.$

Not affordable

Just affordable

Affordable

m/p_1

x_1

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Budget Set and Constraint for Two Commodities

x_2

m/p_2

Budget constraint is
$p_1x_1 + p_2x_2 = m.$

**the collection
of all affordable bundles;
the budget set**

m/p_1

x_1

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Budget Set and Constraint for Two Commodities

x_2

m/p_2

$p_1x_1 + p_2x_2 = m$ is
$x_2 = -(p_1/p_2)x_1 + m/p_2$
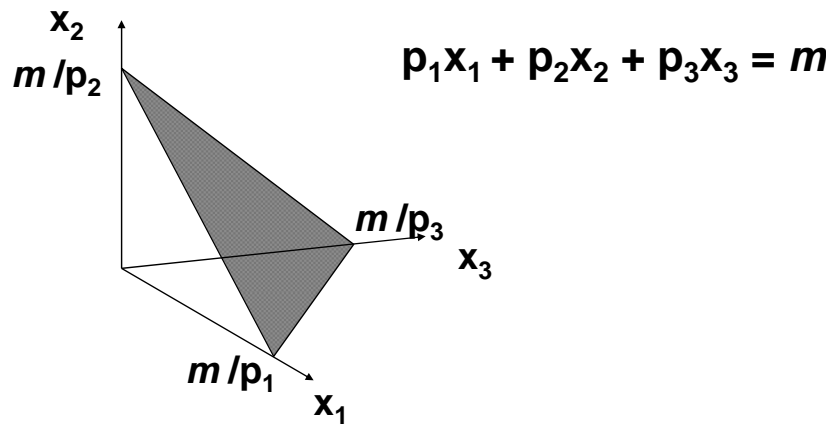so slope is $-p_1/p_2$.

Budget Set

m/p_1

x_1
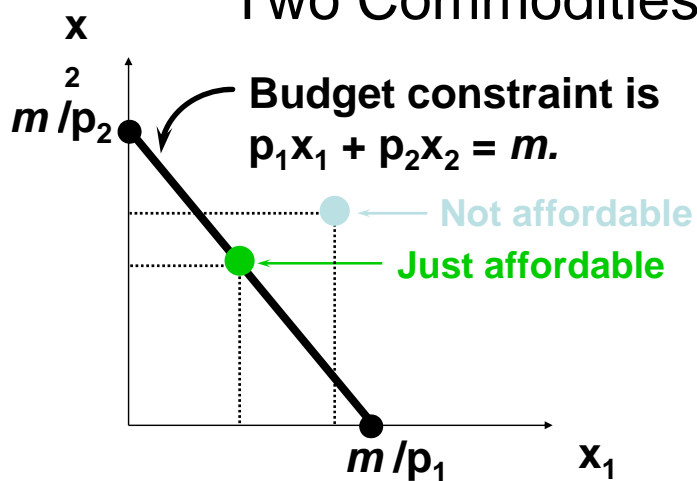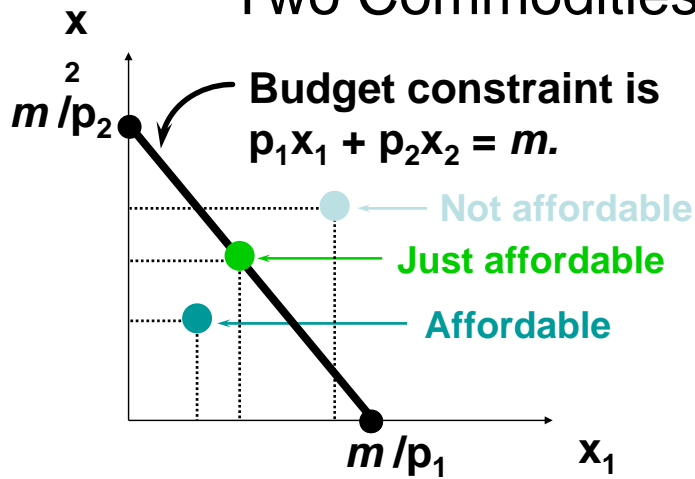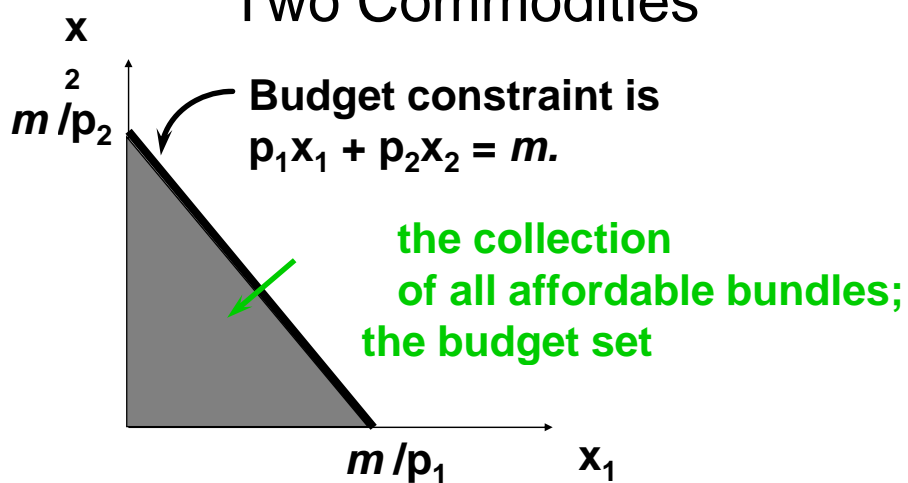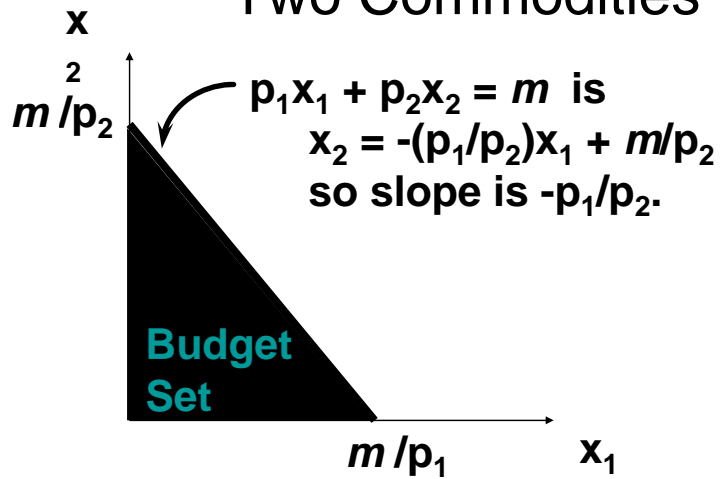
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Budget Constraints

- If n = 3 what do the budget constraint and the budget set look like?

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Budget Constraint for Three Commodities

x_2

m/p_2

$$p_1x_1 + p_2x_2 + p_3x_3 = m$$

m/p_3

x_3

m/p_1

x_1

Budget Set for Three Commodities
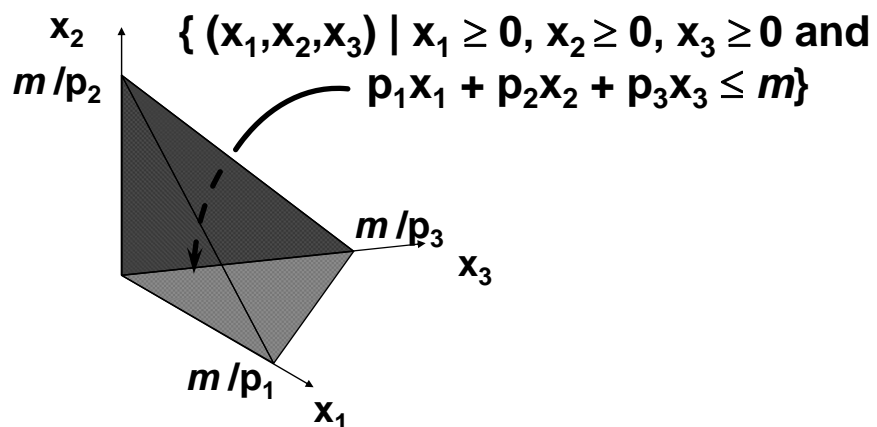
x_2

m/p_2

$\{ (x_1, x_2, x_3) \mid x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$ and
$p_1x_1 + p_2x_2 + p_3x_3 \leq m\}$

m/p_3

x_3

m/p_1

x_1

Budget Constraints

- For n = 2 and x_1 on the horizontal axis, the constraint's slope is $-p_1/p_2$. What does it mean?
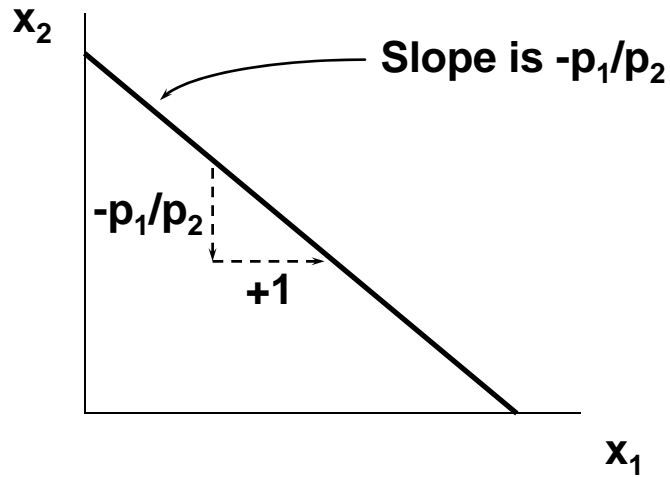
Budget Constraints

- For n = 2 and x_1 on the horizontal axis, the constraint's slope is $-p_1/p_2$. What does it mean?

$$x_2 = - \frac{p_1}{p_2} x_1 + \frac{m}{p_2}$$

- Increasing x_1 by 1 must reduce x_2 by p_1/p_2.
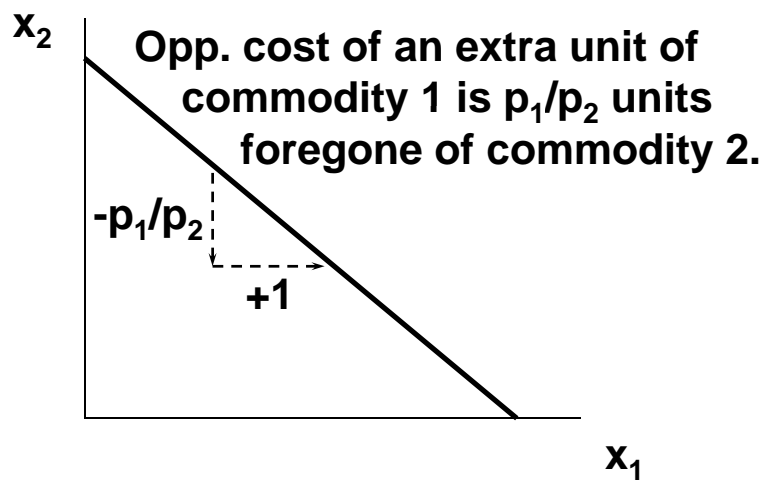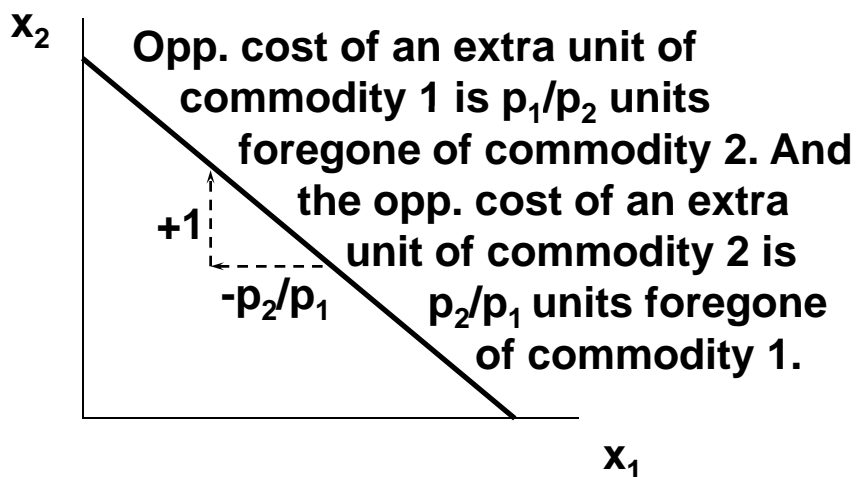
Budget Constraints

x_2

Slope is $-p_1/p_2$

$-p_1/p_2$

$+1$

x_1

Budget Constraints

x_2

Opp. cost of an extra unit of commodity 1 is p_1/p_2 units foregone of commodity 2.

$-p_1/p_2$

$+1$

x_1

Budget Constraints

x_2

Opp. cost of an extra unit of commodity 1 is p_1/p_2 units foregone of commodity 2. And the opp. cost of an extra unit of commodity 2 is p_2/p_1 units foregone of commodity 1.
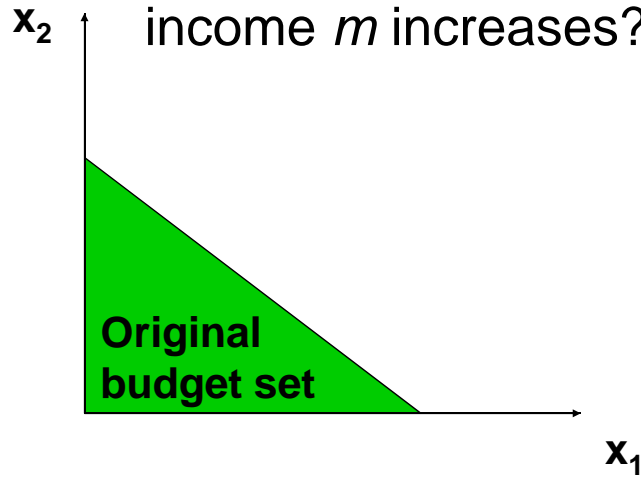
+1

$-p_2/p_1$

x_1

Budget Sets & Constraints; Income and Price Changes

- The budget constraint and budget set depend upon prices and income.

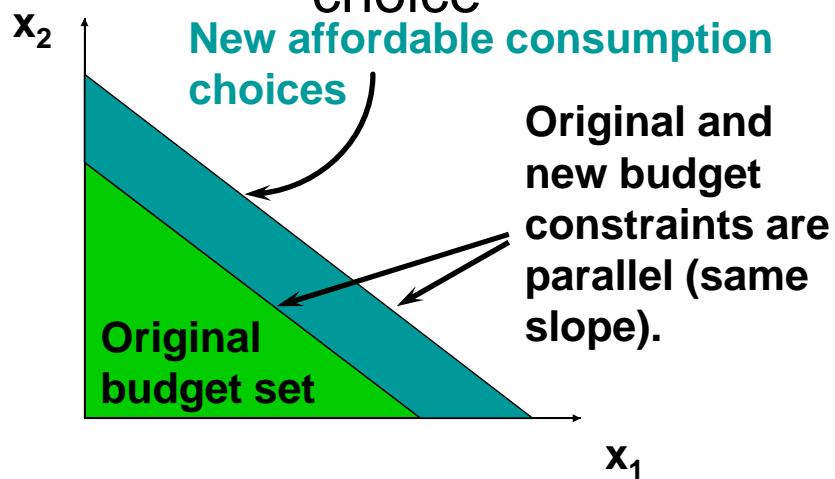- What happens as prices or income change?

How do the budget set and budget constraint change as income *m* increases?
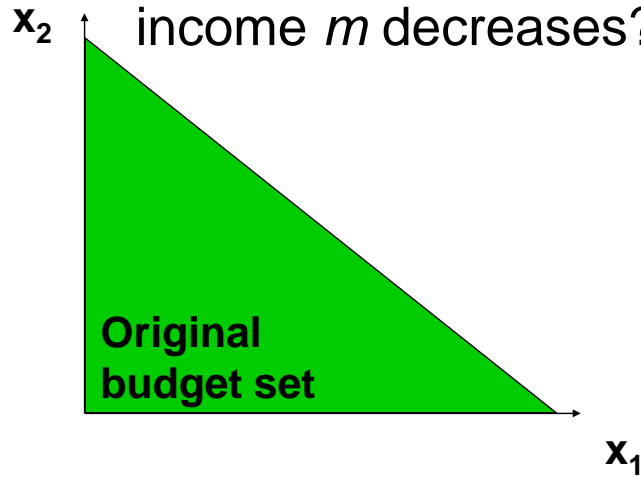
x_2

Original budget set

x_1

Higher income gives more choice

x_2

New affordable consumption choices

Original and new budget constraints are parallel (same slope).

Original budget set

x_1

How do the budget set and budget constraint change as income *m* decreases?

x_2

Original budget set

x_1

How do the budget set and budget constraint change as income *m* decreases?

x_2

Consumption bundles that are no longer affordable.

Old and new constraints are parallel.

New, smaller budget set

x_1
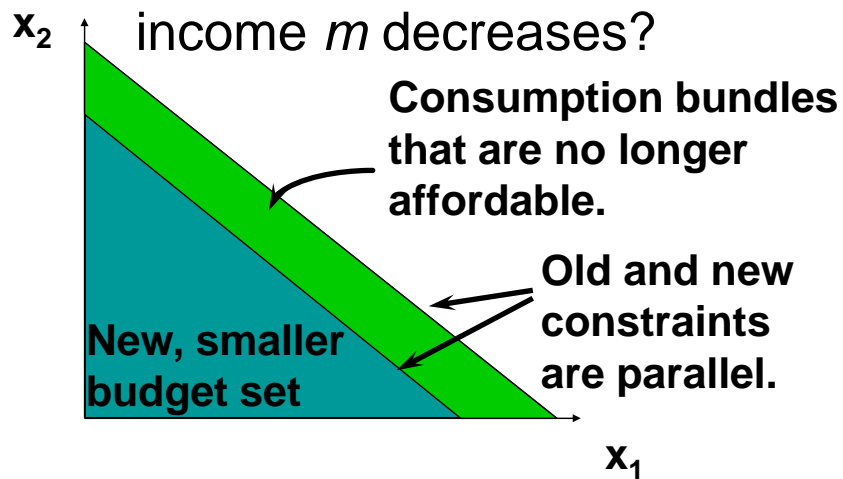
Budget Constraints - Income Changes

- Increases in income *m* shift the constraint outward in a parallel manner, thereby enlarging the budget set and improving choice.

Budget Constraints - Income Changes

- Increases in income *m* shift the constraint outward in a parallel manner, thereby enlarging the budget set and improving choice.

- Decreases in income *m* shift the constraint inward in a parallel manner, thereby shrinking the budget set and reducing choice.

Budget Constraints - Income Changes

- No original choice is lost and new choices are added when income increases, so higher income cannot make a consumer worse off.

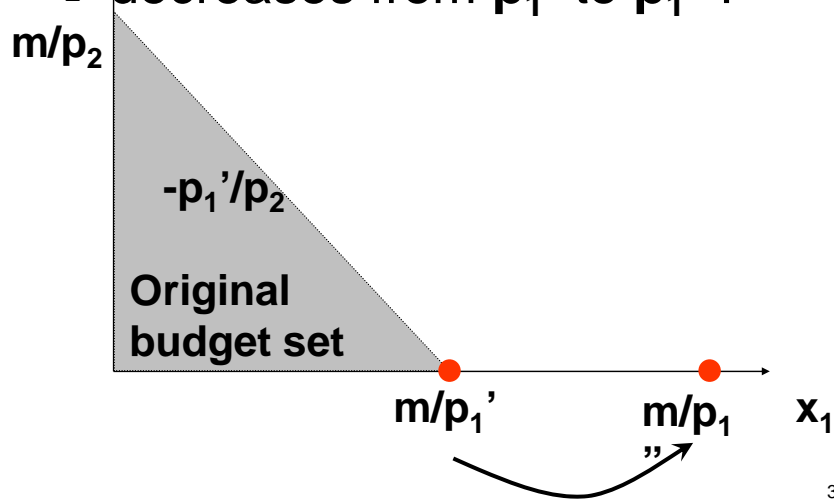- An income decrease may (typically will) make the consumer worse off.

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Budget Constraints - Price Changes

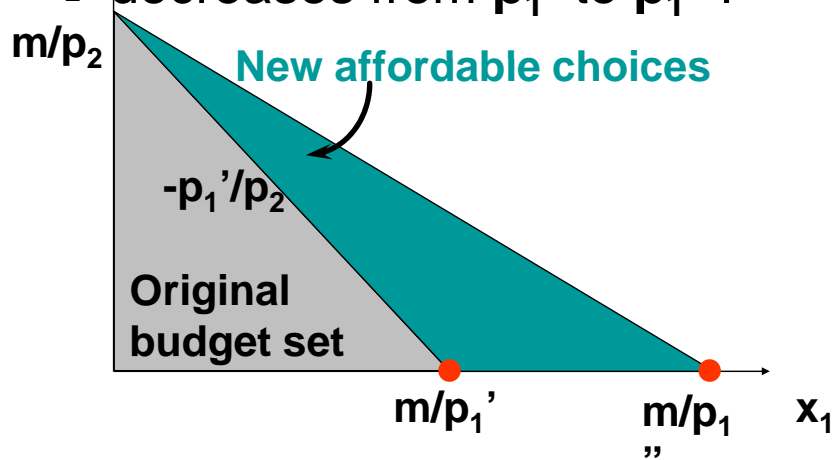- What happens if just one price decreases?
- Suppose p_1 decreases.

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How do the budget set and
budget constraint change as p_1
x_2 decreases from p_1' to p_1''?

m/p_2

$-p_1'/p_2$

**Original
budget set**

m/p_1' m/p_1 x_1
                  ''

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How do the budget set and
budget constraint change as p_1
x_2 decreases from p_1' to p_1''?

m/p_2 **New affordable choices**

$-p_1'/p_2$

**Original
budget set**

m/p_1' m/p_1 x_1
                  ''

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How do the budget set and budget constraint change as p_1 decreases from p_1' to p_1''?

x_2

m/p_2

New affordable choices

Budget constraint pivots; slope flattens from $-p_1'/p_2$ to $-p_1''/p_2$

$-p_1'/p_2$

Original budget set

$-p_1''/p_2$

m/p_1'

m/p_1''

x_1
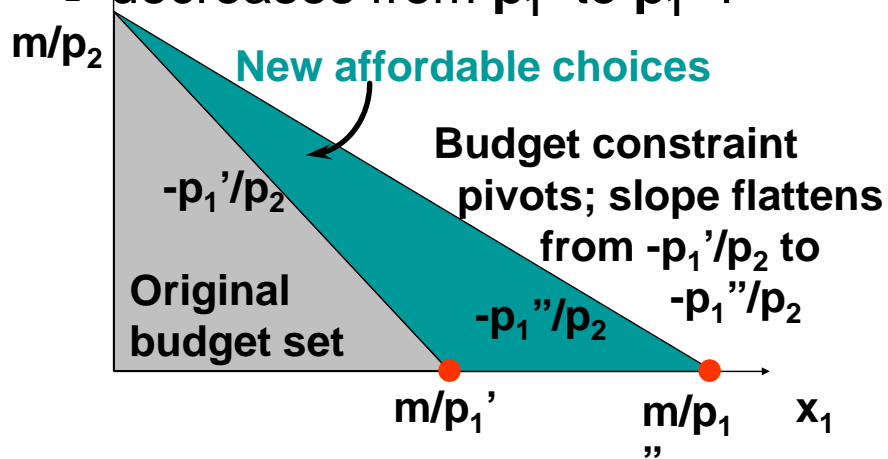
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Budget Constraints - Price Changes

- Reducing the price of one commodity pivots the constraint outward.
- No old choice is lost and new choices are added, so reducing one price cannot make the consumer worse off.

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Budget Constraints - Price Changes

- Similarly, increasing one price pivots the constraint inwards, reduces choice and may (typically will) make the consumer worse off.

Uniform *Ad Valorem* Sales Taxes

- An *ad valorem* sales tax levied at a rate of 5% increases all prices by 5%, from p to $(1+0 \cdot 05)p = 1 \cdot 05p$.

- An *ad valorem* sales tax levied at a rate of *t* increases all prices by *t*p from p to $(1+t)$p.

- A uniform sales tax is applied uniformly to all commodities.

Uniform *Ad Valorem* Sales Taxes

- A uniform sales tax levied at rate t changes the constraint from

$$p_1x_1 + p_2x_2 = m$$

to

$$(1+t)p_1x_1 + (1+t)p_2x_2 = m$$

Uniform *Ad Valorem* Sales Taxes

- A uniform sales tax levied at rate t changes the constraint from
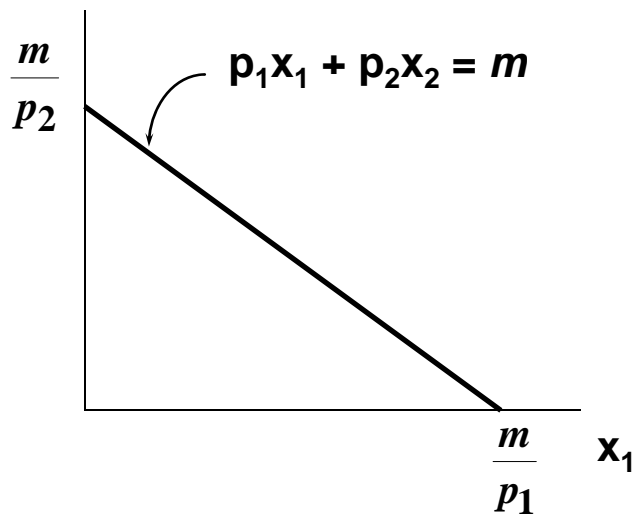
$$p_1x_1 + p_2x_2 = m$$

to

$$(1+t)p_1x_1 + (1+t)p_2x_2 = m$$
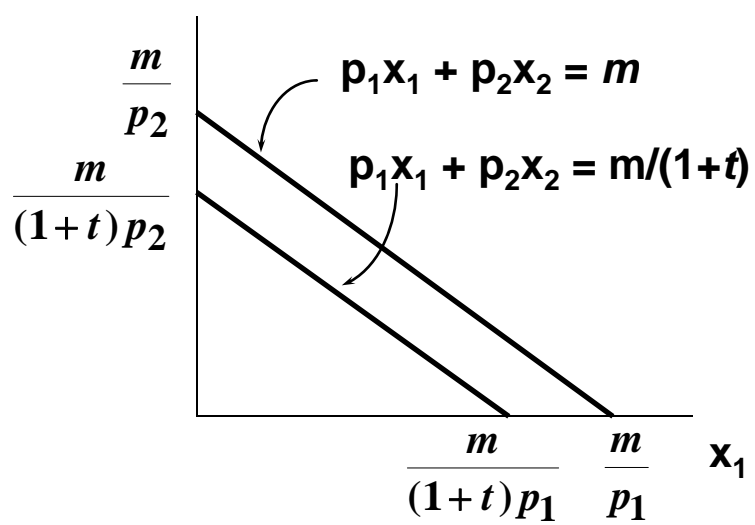
i.e.

$$p_1x_1 + p_2x_2 = m/(1+t).$$

Uniform *Ad Valorem* Sales Taxes

x_2

$\dfrac{m}{p_2}$

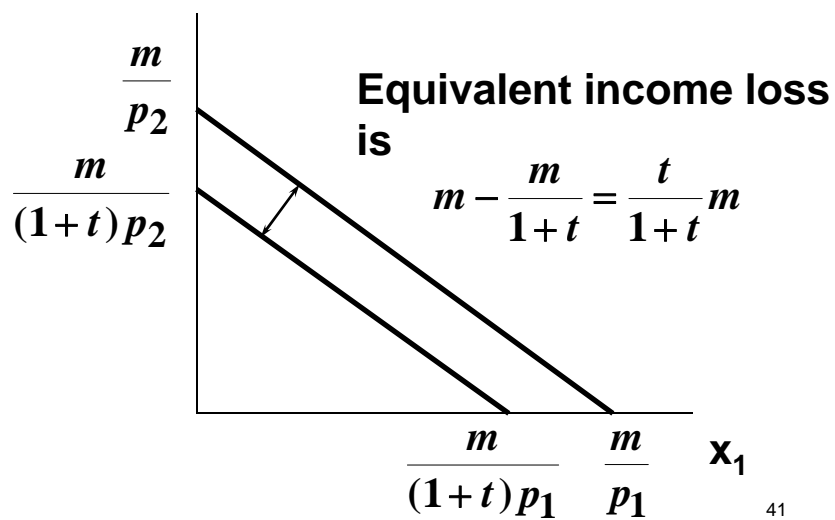$p_1x_1 + p_2x_2 = m$

$\dfrac{m}{p_1}$ x_1

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Uniform *Ad Valorem* Sales Taxes

x_2

$\dfrac{m}{p_2}$

$\dfrac{m}{(1+t)\,p_2}$

$p_1x_1 + p_2x_2 = m$

$p_1x_1 + p_2x_2 = m/(1+t)$

$\dfrac{m}{(1+t)\,p_1}$ $\dfrac{m}{p_1}$ x_1

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Uniform *Ad Valorem* Sales Taxes

x_2

$\dfrac{m}{p_2}$

$\dfrac{m}{(1+t)p_2}$

Equivalent income loss is

$$m - \frac{m}{1+t} = \frac{t}{1+t}m$$

$\dfrac{m}{(1+t)p_1}$ $\dfrac{m}{p_1}$ x_1

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Uniform *Ad Valorem* Sales Taxes

x_2

$\dfrac{m}{p_2}$

$\dfrac{m}{(1+t)p_2}$

A uniform *ad valorem* sales tax levied at rate *t* is equivalent to an income tax levied at rate $\dfrac{t}{1+t}.$

$\dfrac{m}{(1+t)p_1}$ $\dfrac{m}{p_1}$ x_1

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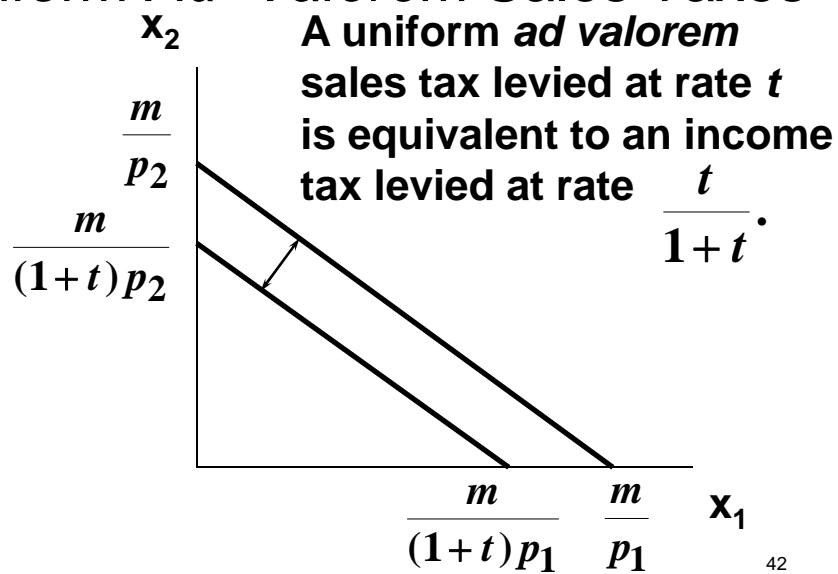
The Food Stamp Program

- Food stamps are coupons that can be legally exchanged only for food.

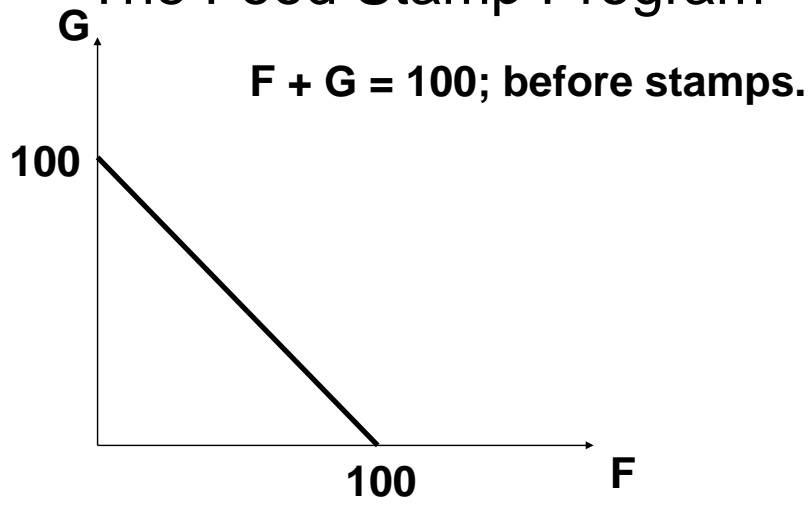- How does a commodity-specific gift such as a food stamp alter a family's budget constraint?

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The Food Stamp Program

- Suppose m = $100, p_F = $1 and the price of "other goods" is p_G = $1.
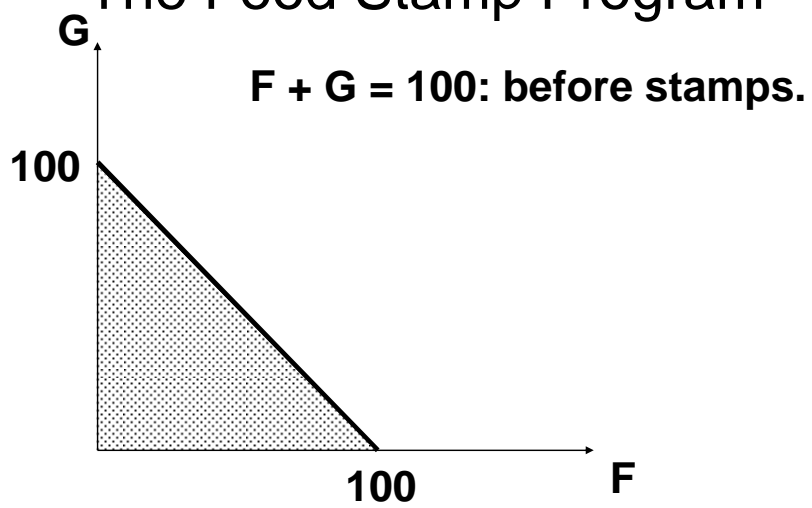
- The budget constraint is then
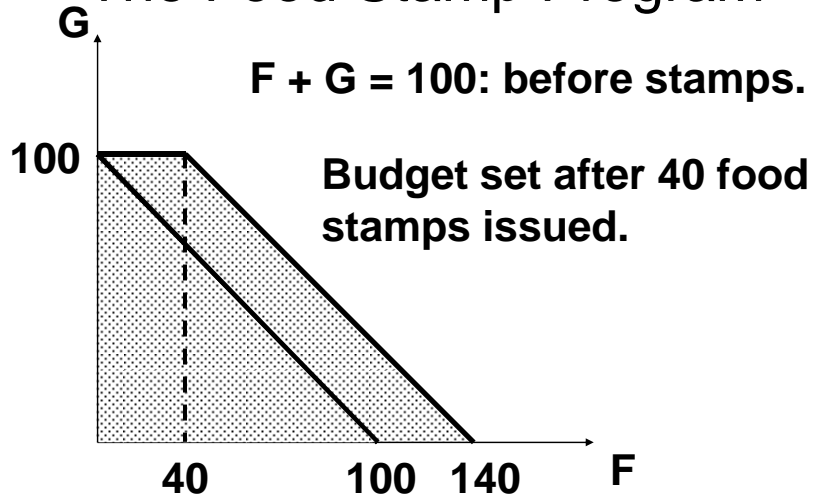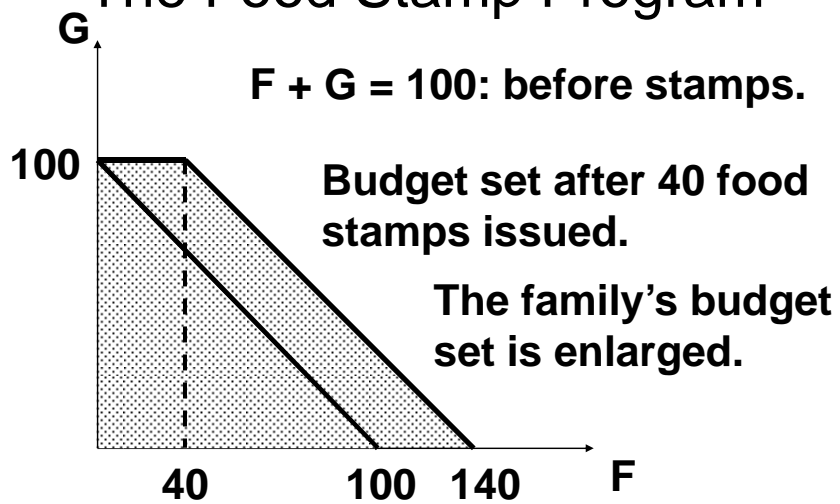 $$F + G = 100.$$

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The Food Stamp Program

G

F + G = 100; before stamps.

100

100

F

45

The Food Stamp Program

G

F + G = 100: before stamps.

100

100

F

46

The Food Stamp Program

G

F + G = 100: before stamps.

100

Budget set after 40 food stamps issued.

40　　**100**　**140**　　**F**

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The Food Stamp Program

G

F + G = 100: before stamps.

100

Budget set after 40 food stamps issued.

The family's budget set is enlarged.

40　　**100**　**140**　　**F**

48

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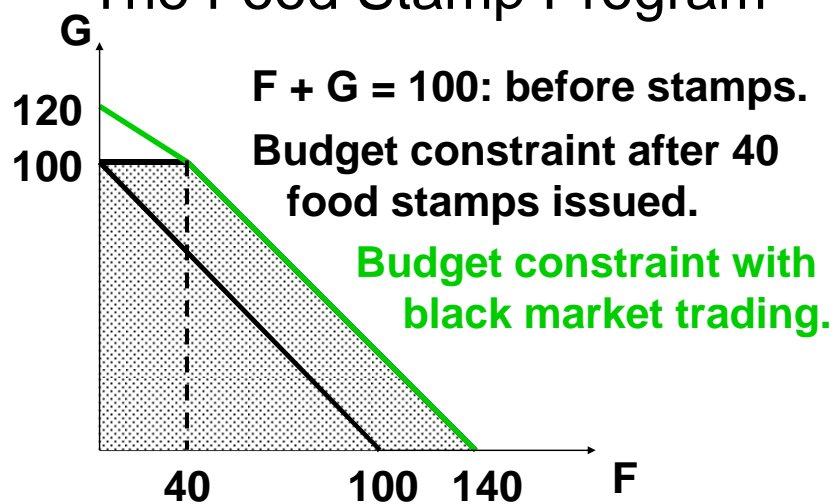
The Food Stamp Program

- What if food stamps can be traded on a black market for $0.50 each?

The Food Stamp Program

F + G = 100: before stamps.

Budget constraint after 40 food stamps issued.

Budget constraint with black market trading.

G — 120, 100

F — 40, 100, 140

The Food Stamp Program

G

120

100

F + G = 100: before stamps.

Budget constraint after 40 food stamps issued.

Black market trading makes the budget set larger again.

40   **100   140**   **F**
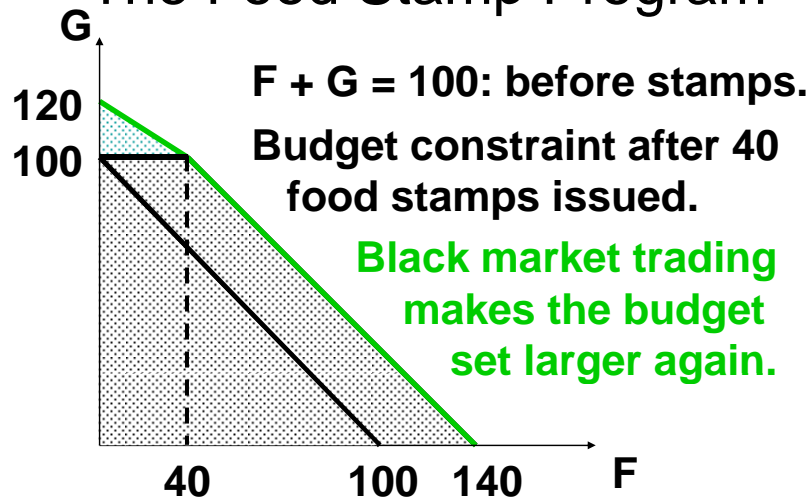
Budget Constraints - Relative Prices

- "Numeraire" means "unit of account".

- Suppose prices and income are measured in dollars. Say p_1=$2, p_2=$3, m = $12.

- Then the constraint is
$$2x_1 + 3x_2 = 12.$$

Budget Constraints - Relative Prices

- If prices and income are measured in cents, then $p_1=200$, $p_2=300$, $m=1200$ and the constraint is

$$200x_1 + 300x_2 = 1200,$$

the same as

$$2x_1 + 3x_2 = 12.$$

- Changing the numeraire changes neither the budget constraint nor the budget set.

Budget Constraints - Relative Prices

- The constraint for $p_1=2$, $p_2=3$, $m=12$

$$2x_1 + 3x_2 = 12$$
$$1.x_1 + (3/2)x_2 = 6,$$

the constraint for $p_1=1$, $p_2=3/2$, $m=6$.

- Setting $p_1=1$ makes commodity 1 the numeraire and defines all prices relative to p_1;

- *e.g.* 3/2 is the price of commodity 2 relative to the price of commodity 1.

Budget Constraints - Relative Prices

- Any commodity can be chosen as the numeraire without changing the budget set or the budget constraint.

Budget Constraints - Relative Prices

- $p_1=2$, $p_2=3$ and $p_3=6$ $\Rightarrow$
- price of commodity 2 relative to commodity 1 is 3/2,
- price of commodity 3 relative to commodity 1 is 3.
- Relative prices are the rates of exchange of commodities 2 and 3 for units of commodity 1.

Shapes of Budget Constraints

- Q: What makes a budget constraint a straight line?
- A: A straight line has a constant slope and the constraint is

$$p_1 x_1 + \ldots + p_n x_n = m$$

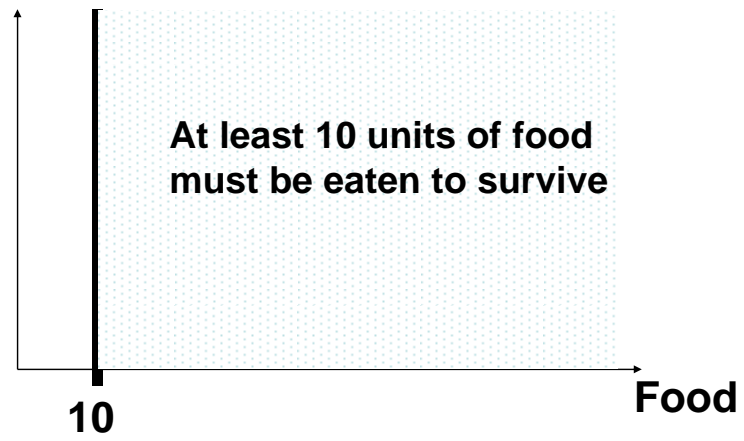- so if prices are constants then a constraint is a straight line.

More General Choice Sets

- Choices are usually constrained by more than a budget; e.g. time constraints and other resources constraints.

- A bundle is available only if it meets every constraint.

More General Choice Sets

Other Stuff

At least 10 units of food must be eaten to survive

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Food

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