Chapter Five

Choice

Economic Rationality

- The principal behavioral postulate is that a decisionmaker chooses its most preferred alternative from those available to it.
- The available choices constitute the choice set.
- How is the most preferred bundle in the choice set located?

Rational Constrained Choice

Consider well-behaved preferences as :

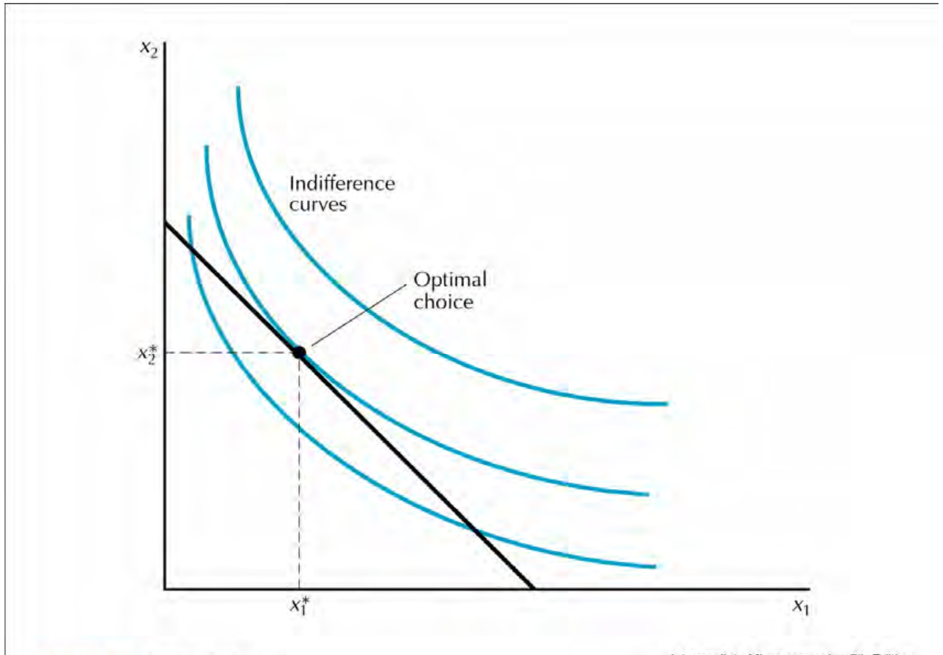
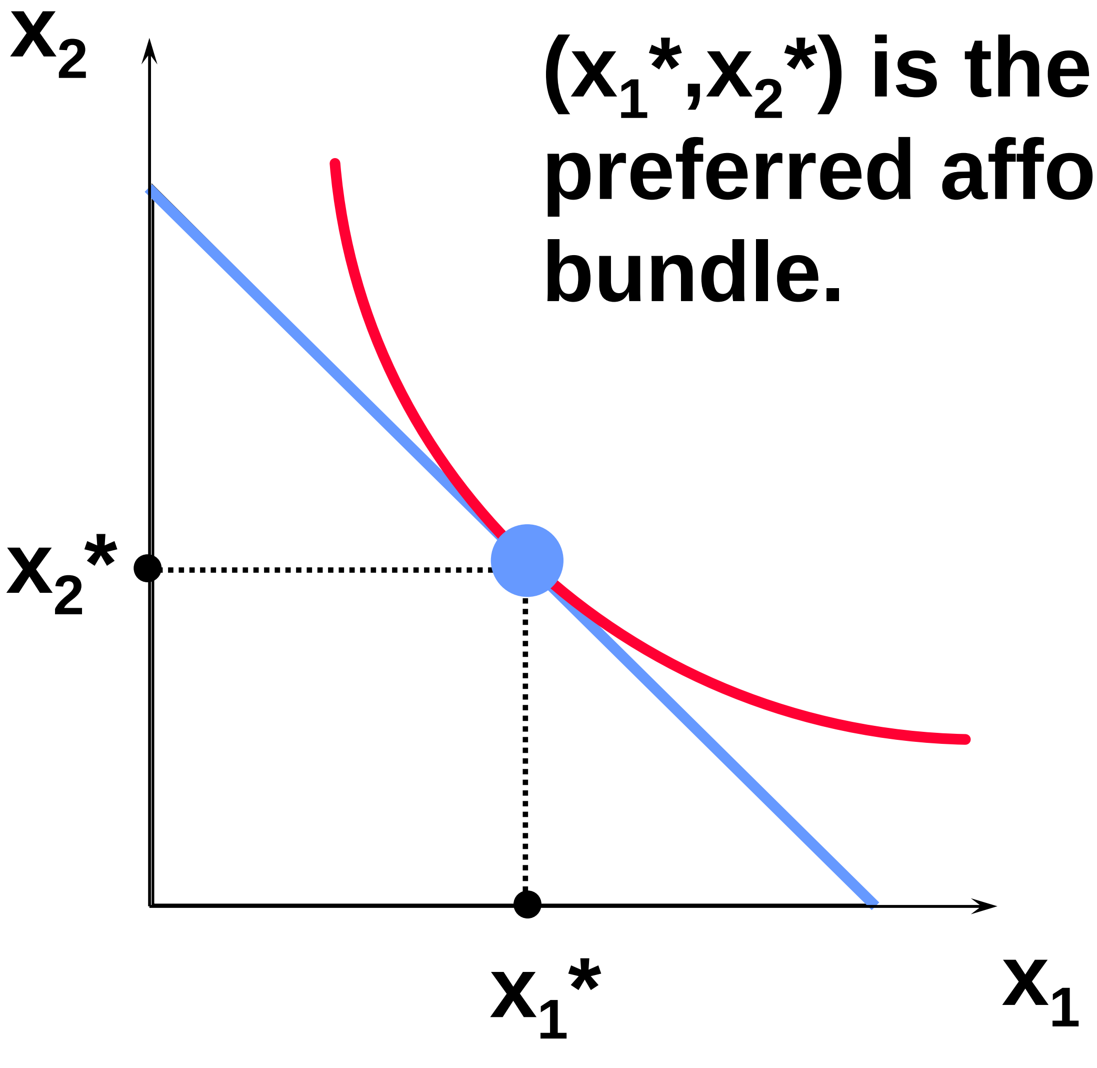


FIGURE 5.1 Optimal choice

We restrict attention to the bundles on the budget line (more is better).

We must have the optimal bundle where the budget line "just" touches the indifference curve, i.e. where the budget line is a tangent to the indifference curve. Discuss!

Rational Constrained Choice



(x_1^*,x_2^*) is the most preferred affordable bundle.

Rational Constrained Choice

- Recall the slope of the budget line was $-p_1/p_2$.
- This was the rate at which the market traded with the consumer
- But also recall the slope of the indifference curve was the **MRS**
- This was the valuation that the consumer had of giving up some of the goods for some more of the other

Rational Constrained Choice

- Moreover recall from last chapter that MRS was

$$MRS = \frac{dx_2}{dx_1} = -\frac{\partial U/\partial x_1}{\partial U/\partial x_2} = -\frac{MU_1}{MU_2}$$

Rational Constrained Choice

- Thus our optimal choice must satisfy:

$$MRS = \frac{dx_2}{dx_1} = -\frac{\partial U / \partial x_1}{\partial U / \partial x_2} = -\frac{MU_1}{MU_2} = -\frac{P_1}{p_2}$$

What is the intuition?

Intuition for Optimal Condition

- The MRS measure the rate we are willing to trade some good for another without loss in utility, i.e. that might be 1 say.

- Then, we are willing to give up 1 good if we get 1 good of the other, or we might as well stay put, since we are indifferent.

- Say now the market, which is about prices, is willing to trade at the same price.
- E.g, $-p_1/p_2$. = (-4/4)=(-1/1).

- Then, since I don't value (in terms of utility) to have something more of one good and something less of the other differently from the market, I can't get it better and might as well stay put.
- But if that´s not the case...

Intuition for Optimal Condition

- then there is scope for improvement.
- Why, assume $-p_1/p_2$. = (4/2)=(2/1),
- then in terms of my valuation, x_2 is inexpensive and of course equivalently x_1 is expensive.
- Then I want to trade with the market and increase my consumption of x_2, by giving up some x_1.
- An optimizing agent will continue this trade up to the point of were she can't get it better i.e. when

$$MRS = \frac{dx_2}{dx_1} = -\frac{\partial U/\partial x_1}{\partial U/\partial x_2} = -\frac{MU_1}{MU_2} = -\frac{P_1}{p_2}$$

How to find the optimal x

- We have 2 conditions in the two goods case:

$$1.\ MRS = \frac{dx_2^*}{dx_1^*} = -\frac{\partial U/\partial x_1^*}{\partial U/\partial x_2^*} = -\frac{MU_1}{MU_2} = -\frac{P_1}{p_2}$$

And

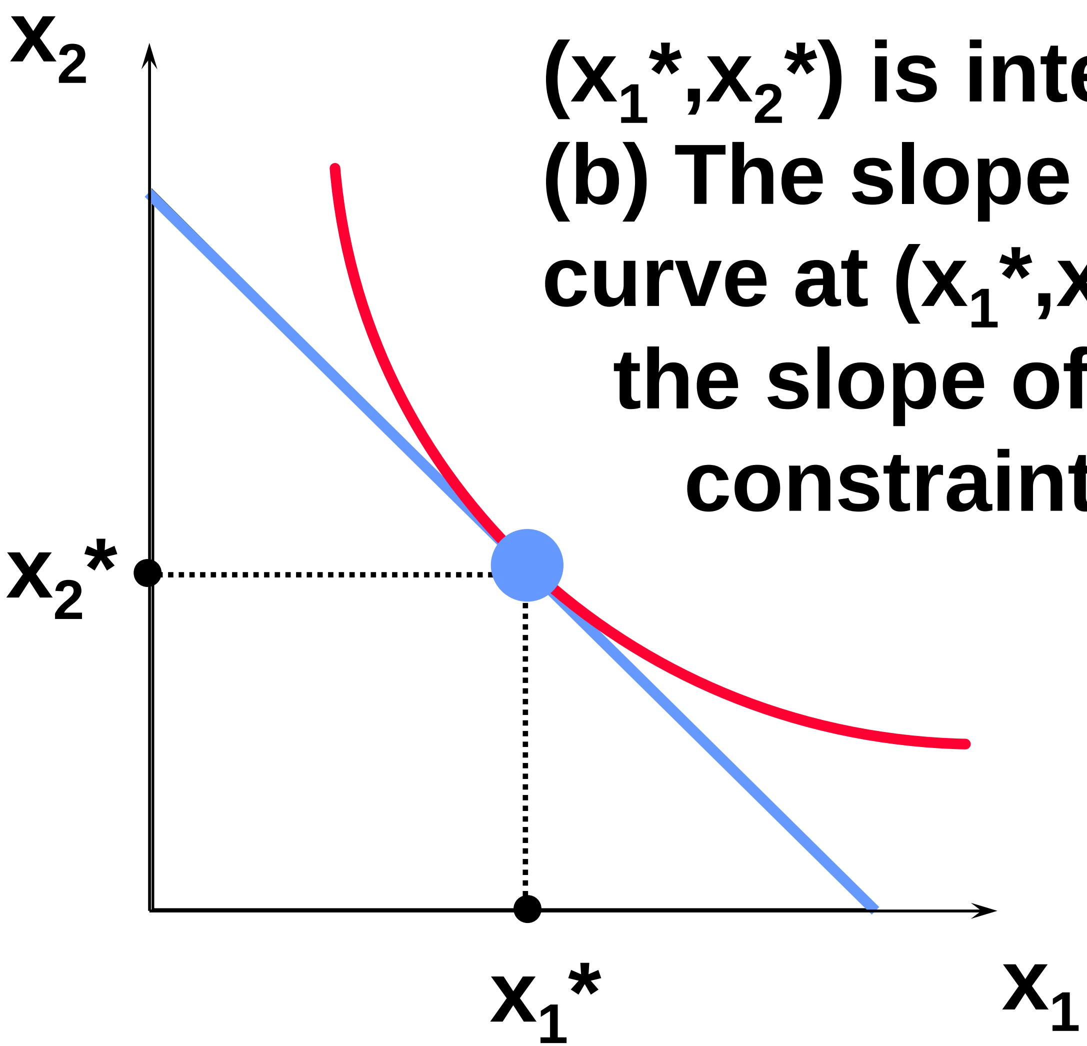
We know we must consume on the budget line, since more is better. Thus:

$$2.\ p_1 x_1^* + p_2 x_2^* = m.$$

How to find the optimal x

- Why stars on the on the x?
- When these two conditions hold, the stars indicate the solution for x_1 and x_2
- Ordinary demands (Marshallian Demand), the solution, will typically be a function and will be denoted by
- $x_1(p_1,p_2,m)$ and $x_2(p_1,p_2,m)$.
- That is typically there uniqely these x_1^* and x_2^* that solves it for given p_1,p_2,m
- Clearly, for new p'_1,p'_2,m' than there are new solutions: $x_1(p'_1,p'_2,m')$ and $x_2(p'_1,p'_2,m')$. Hence these are called functions

Rational Constrained Choice



(x_1^*,x_2^*) is interior .
(b) The slope of the indiff. curve at (x_1^*,x_2^*) equals the slope of the budget constraint.

Computing Ordinary Demands

- How can this information be used to locate (x_1^*, x_2^*) for given p_1, p_2 and m?

Computing Ordinary Demands - a Cobb-Douglas Example.

- Suppose that the consumer has Cobb-Douglas preferences.

Then $$U(x_1, x_2) = x_1^a x_2^b$$

$$\mathbf{MU_1 = \frac{\partial U}{\partial x_1} = a x_1^{a-1} x_2^b}$$

$$\mathbf{MU_2 = \frac{\partial U}{\partial x_2} = b x_1^{a} x_2^{b-1}}$$

Computing Ordinary Demands - a Cobb-Douglas Example.

- So the MRS is

$$MRS = \frac{dx_2}{dx_1} = -\frac{\partial U/\partial x_1}{\partial U/\partial x_2} = -\frac{a x_1^{a-1} x_2^b}{b x_1^a x_2^{b-1}} = -\frac{a x_2}{b x_1}.$$

- At (x_1^*, x_2^*), MRS = $-p_1/p_2$ so

$$-\frac{a x_2^*}{b x_1^*} = -\frac{p_1}{p_2} \Rightarrow x_2^* = \frac{b p_1}{a p_2} x_1^*. \qquad \textbf{(A)}$$

Computing Ordinary Demands - a Cobb-Douglas Example.

- So now we know that

$$x_2^* = \frac{bp_1}{ap_2} x_1^* \qquad \textbf{(A)}$$

$$p_1 x_1^* + p_2 x_2^* = m. \qquad \textbf{(B)}$$

Computing Ordinary Demands - a Cobb-Douglas Example.

- So now we know that

Substitute $$x_2^* = \frac{bp_1}{ap_2} x_1^*$$ **(A)**

$$p_1 x_1^* + p_2 x_2^* = m.$$ **(B)**

and get $$p_1 x_1^* + p_2 \frac{bp_1}{ap_2} x_1^* = m.$$

This simplifies to ….

Computing Ordinary Demands - a Cobb-Douglas Example.

$$x_1^* = \frac{am}{(a+b)p_1}.$$

Substituting for x_1* in

$$p_1 x_1^* + p_2 x_2^* = m$$

then gives

$$x_2^* = \frac{bm}{(a+b)p_2}.$$

Computing Ordinary Demands - a Cobb-Douglas Example.

So we have discovered that the most preferred affordable bundle for a consumer with Cobb-Douglas preferences

$$U(x_1, x_2) = x_1^a x_2^b$$

is

$$(x_1^*, x_2^*) = \left(\frac{am}{(a+b)p_1}, \frac{bm}{(a+b)p_2} \right).$$

Computing Ordinary Demands - a Cobb-Douglas Example.

$$U(x_1, x_2) = x_1^a x_2^b$$