Demand

(Baseret på Varian, chapter 6)

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Optimal Choice

- Consider well-behaved preferences as the picture:

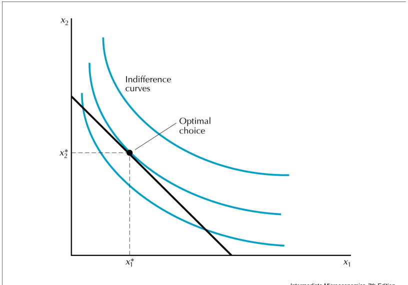

FIGURE 5.1 Optimal choice

- We restrict attention to the bundles on the budget line(more is better).
- We must have the optimal bundle where the budget line "just" touches the indifference curve, i.e. where the budget line is a tangent to the indifference curve,
- Evident from the picture by comparing the different indifference curves.

Optimal Condition

- Now we graphically have shown the solution to a well-behaved problem. Recall the budget line had the slope of -$\frac{p_j}{p_i}$. Moreover recall the slope of the indifference curve which was called the

$$MRS = -\frac{\frac{\partial u(\mathbf{x})}{\partial x_i}}{\frac{\partial u(\mathbf{x})}{\partial x_j}} = -\frac{MU_{x_i}}{MU_{x_j}},$$

- Hence in optimum we must have that

$$MRS = \frac{\frac{\partial u(\mathbf{x})}{\partial x_i}}{\frac{\partial u(\mathbf{x})}{\partial x_j}} = \frac{p_i}{p_j},$$

Back to the general formulation of Marshallian Demand

- We call $\mathbf{x}(\mathbf{p}, m)$ the (Marshallian) demand function.
- Note a function $f(\mathbf{x})$ is homogenous of degree k if $f(t\mathbf{x}) = t^k f(\mathbf{x})$
- Thus, homogenous of degree 0 is that $f(t\mathbf{x}) = f(\mathbf{x})$ and of degree 1 is $f(t\mathbf{x}) = tf(\mathbf{x})$ and so on.
- Note $\mathbf{x}(\mathbf{p}, m)$ is homogenous of degree 0 in $(\mathbf{p}, m)$. Show on the board

Comparative Statics of the Demand Function

- Comparative statics is how something responds to change in the environment.
  - Specifically, how does the demand functions $\mathbf{x}(\mathbf{p}, m)$ change to changes in one of the prices or m.(before vs. after),everything else constant.

Definitions

- A good i is **normal** if

$$\frac{\partial x_i(\mathbf{p}, m)}{\partial m} \geq 0$$

- I.e. we consume more of it if wealth increases, everything else equal. Can you think of some example in the real world? Show with Cobb-Douglas?
- A good i is **inferior** if

$$\frac{\partial x_i(\mathbf{p}, m)}{\partial m} < 0$$

- I.e. we consume less of it if wealth increases, everything else equal. Can you think of some example? Can all goods be inferior?
- Note the sign might depend of how rich we are. For some m we might treat noodles as normal, but not if we get richer.

Definitions

- Our **income expansion path**, we deduce by varying wealth or income, m, and see how this changes our optimal consumption **bundle**
  - If both goods are normal, naturally this will be upward sloping.
- If we just focus on one of the goods, we call it the **Engel curve** and the slope is given by the partial derivative.

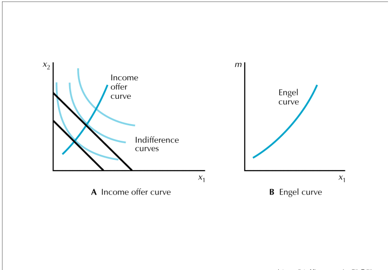

FIGURE 6.3 How demand changes as income changes

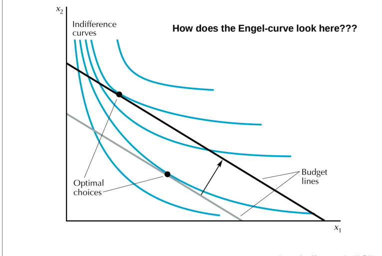

FIGURE 6.2 An inferior good

Definitions. Introducing Elasticity.

- If the income expansion path bends towards one good , i.e. one good is increased by larger proportion than the increase in income, we call the good a **luxury good** and the other we call a **necessary good**. Note however if it bends to much we have a inferior good. (Show on the board)
- Formally, this can be expressed in terms of **income elasticity**
- The (income) elasticity of demand is not depending on the units we measure quantity or prices in (the slope is, why?)
- The income elasticity answers the question **how many percents do demand increase (decrease), when income is increased one percent**.

$$Income\ elasticity\ of\ demand = \frac{\%\ change \text{ in quantity}}{\%\ change \text{ in income}}$$

Definitions. Introducing Elasticity.

- This can be written

$$\epsilon_m \equiv \frac{\frac{\partial x_i(\mathbf{p},m)}{x_i}}{\frac{\partial m}{m}} = \frac{\partial x_i(\mathbf{p},m)}{\partial(m)} \frac{m}{x_i}$$

- A good i is a **luxury** good if

$$\epsilon_m > 1$$

- A good i is a **necessary** good if

$$0 \leq \epsilon_m \leq 1$$

- Both of these types must be **normal** goods. Why?
- A good i is a **inferior** good if

$$\epsilon_m < 0$$

Price Changes

- A **normal and a inferior good** i also have(which we shall see clearly in later lectures)

$$\frac{\partial x_i(\mathbf{p}, m)}{\partial p_i} \leq 0$$

- An obscure fact is that we can have a **Giffen good**, where

$$\frac{\partial x_i(\mathbf{p}, m)}{\partial p_i} > 0$$

- Why is this obscure?

Normal and Giñen Goods

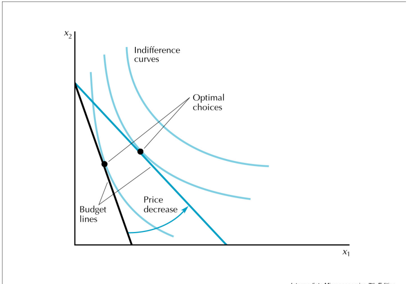

FIGURE 6.9 An ordinary good

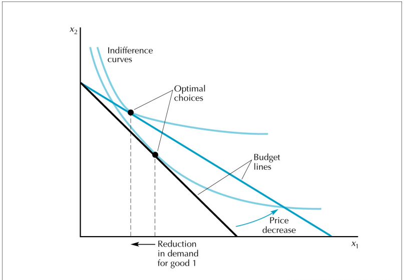

FIGURE 6.10 A Giffen good

- Can you think of any real life examples of Giffen goods? New article claiming it can exist: See http://www.cid.harvard.edu/cidwp/pdf/148.pdf
- What if we plot the optimal choice if one of the prices changes (the price offer curve). Let me do that on the board.

Complements and Substitutes Again

- Now we can also formulate the conditions for compliments and substitutes
  - good i is **substitute** for good j if

$$\frac{\partial x_i(\mathbf{p}, m)}{\partial p_j} \geq 0$$

  - good i is **complement** for good j if

$$\frac{\partial x_i(\mathbf{p}, m)}{\partial p_j} < 0$$

The Inverse Demand Function

- If can solve $x_i(\mathbf{p}, m)$ for p_i as $p_i(\mathbf{p}, m, x_i)$ and plot it against x_i we get the normal **inverse demand function**
- Example recall from the Cobb-Douglas Example that $x_1^*(p_1, p_2, m) = \frac{\alpha m}{p_1}$.Thus, if we solve for p_1 we get

$$p_1 = \frac{\alpha m}{x_1^*(p_1, p_2, m)}$$

- Plotting this as a function of x_i gives normally the downward sloping **inverse demand function** that we have seen so many times (what happens if there is a Giffen good?)
- Note, when plotting this, we hold everything else constant except for p_1 and x_1.

The Inverse Demand Function. Example.

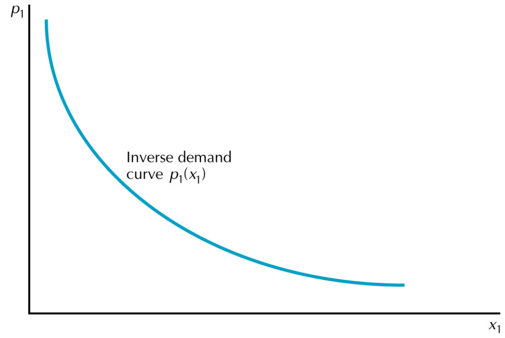

FIGURE 6.15 Inverse demand curve