AALBORG UNIVERSITY
DENMARK

GEOMETRIC AND TOPOLOGICAL METHODS IN COMPUTER SCIENCE - GETCO 2015

ABSTRACTS

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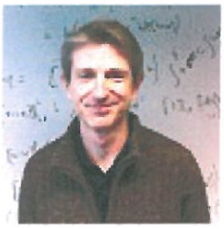

Eric Goubault, École Polytechnique, Paris

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Krzysztof Ziemianski

Spaces of directed paths on semi-cubical sets

One of objects used for modeling parallel computing are Higher Dimensional Automata (HDA). A special case of HDA's are PV-programs which use Dijkstra's semaphores for coordination of executions of processes.

The space of all possible executions of a Higher Dimensional Automaton is the space of directed paths between two points on the underlying cubical set. I will present a construction of a CW-complex which is homotopy equivalent to such a space. This construction satisfies certain minimality condition which makes it useful for direct calculations. I will also discuss the relationship of this construction with other models of directed path spaces.

Philippe Malbos

Oriented Syzygies for Monoids

A coherent presentation of a monoid is an extension of a presentation of this monoid by oriented syzygies, making a natural cellular complex associated to the presentation contractible. Such extended presentation are used for representations of monoids, in particular, it is a way to describe actions of monoids on categories.
Moreover, a coherent presentation provides the first step in the computation of a categorical cofibrant replacement of a monoid.

We will present a rewriting method to construct coherent presentations that we will illustrate on two families of monoids: Artin monoids and Plactic monoids. In the case of Artin monoids, we show that the usual presentation defined by Artin, using braid relations, can be completed in a coherent presentation that we give in an explicit way.

Ulrich Bauer

Induced Matchings and the Algebraic Stability of Persistence Barcodes

We define a simple, explicit map sending a morphism $f : M \to N$ of pointwise finite dimensional persistence modules to a matching between the barcodes of M and N. Our main result is that, in a precise sense, the quality of this matching is tightly controlled by the lengths of the longest intervals in the barcodes of ker f and coker f.

As an immediate corollary, we obtain a new proof of the algebraic stability theorem for persistence barcodes, a fundamental result in the theory of persistent homology. In contrast to previous proofs, ours shows explicitly how a δ-interleaving morphism between two persistence modules induces a δ-matching between the barcodes of the two modules. Our main result also specializes to a structure theorem for submodules and quotients of persistence modules, and yields a novel "single-morphism" characterization of the interleaving relation on persistence modules.

Pawel Dlotko

Applied computational topology, where we should go now?

In this talk I will touch two aspects of computational topology:
1) A problem of discretization: I will discuss how to rigorously compute a complex having the same homotopy type as a level set of a continuous function and how to rigorously compute persistent homology of a continuous function restricted to a compact domain.
2) A problem of statistical manipulations of a family of persistence intervals.
Both of those problems have practical origin. Computation of a level set of a function is used in mesh generation software. Scientists working in numerical analysis may want to compute persistence of a continuous function instead of a point cloud and get mathematically rigorous results. At the end, scientists doing topological data analysis often have to manipulate large families of persistence diagrams and draw some conclusions based on them. In this talk I will show how this can be practically done. It time permits, I will present some way to discuss distributed implementation of an algorithm to compute persistent homology.

Sergio Rajsbaum

Introduction to distributed computing analysis using combinatorial topology

An introductory overview to the recently published book
"Distributed Computing Through Combinatorial Topology by
Maurice Herlihy, Dmitry Kozlov, and Sergio Rajsbaum
is presented.

Armando Castaneda

Computing Independent Sets in an Asynchronous Environment

We consider an independent set task for a set of asynchronous crash-prone processes that have to output an independent set of a graph G, as large as possible. Processes communicate through a read/write shared-memory. Each process starts with a preference for a vertex in G, communicates with other processes, and decides on an output vertex. The output vertices are distinct, and no two output vertices belong to the same edge. Furthermore, if all initial preferences are distinct, and form an independent set of G, then the output vertex of each process is equal to its initial preference. The independent set number of G is the largest number of processes n that can solve the task on G.

We propose a wait-free distributed independent set algorithm (tolerating any number of process crashes), and show that it has optimal independent set number, among static independent set algorithms. Also, we prove that for many graphs, no independent set algorithm can do better. In a static independent set algorithm there is an independent set I such that in every execution, if a process does not decide its initial input, it decides a vertex in I. We use topological techniques to prove our impossibility results.

Rick Jardine

Path categories and algorithms

The theory of path categories and path 2-categories for finite oriented cubical and simplicial complexes will be reviewed. There is an algorithm for computing the path category P(K) of a finite complex K, which is based on its path 2-category. This "2-category algorithm" will be displayed, and complexity reduction methods for the algorithm will be discussed.

The 2-category algorithm works well only for toy examples. The size of the path category P(K) can be an exponential function of the size of K.
The algorithm has so far resisted parallelization.

One wants combinatorial local to global methods for addressing examples that are effectively infinite. The time variable gives a coarse measure of distance between states, but it is probably only locally defined in the right big picture. The existence of paths between states is an issue in large examples.

Sanjeevi Krishnan

Dynamic Sensor Networks (joint work with Rob Ghrist)

A class of pursuit-evasion games, where an evader tries to avoid detection by a time-evolving sensed space, is considered. Earlier results give homological criteria for evasion that are necessary or sufficient, but not both. We give a necessary and sufficient ordered cohomological criterion for evasion in the general case. The main idea is to refine the cohomology of a coverage region with a positive cone encoding orientation, refine the homology of the coverage gaps with a positive cone encoding time, and prove a positive Alexander Duality in homological degree 1. Positive cohomology, the limit of a sheaf of local positive cohomology semigroups on the real number line, can be computed as a linear programming problem. We demonstrate such a calculation for a prototypical case of evasion that eludes ordinary homological criteria.

Neza Mramor-Kosta

On perfect discrete Morse functions

A perfect discrete Morse function on a cell complex has the minimal possible number of critical elements. Existence of such functions is important since they provide efficient algorithms for computing topological invariants. In the talk obstructions for the existence of perfect discrete Morse functions on regular cell complexes will be discussed and some algorithms for obtaining them will be presented.

Primoz Skraba

Sheaves and Global Sections

Persistent homology has been successful for studying one-parameter families of spaces (or pairs of spaces). The direct generalization to more general families of spaces (e.g. multi-parameter families of spaces, networks of spaces, etc) has proven difficult for algebraic reasons. Informally, sheaves can be thought of as attaching local data to open sets of a space If the sheaf axioms (which are local) are satisfied then we can look for global sections of the data over the space, or rather features in the data which are globally consistent.
The benefit of this type of approach is that it only requires local models and can be extended to work over a wide-range of spaces (including networks).

In this talk I will describe computational aspects of computing global sections and different approaches to encoding the local structure.

Marian Mrozek

Morse-Forman-Conley theory for combinatorial multivector fields

In late 90' R. Forman defined a combinatorial vector field on a CW complex and presented a version of Morse theory for acyclic combinatorial vector fields. He also studied combinatorial vector fields without acyclicity assumption, studied its chain recurrent set and proved Morse inequalities in this setting.
In this talk we consider a generalized concept of combinatorial multivector field and present an extension of the Morse-Forman theory towards the Conley index theory.

This is research in progress.

Hubert Wagner

Generalized similarity measure for texts

We present a new direction in topological analysis of text documents.
Specifically, we focus on generalizing the notion of similarity measure from pairs of documents to arbitrary tuples. Our new definition is geometric in nature and falls into the category of Bregman divergences. It can be a basis for topological analysis e.g.using persistent homology. First, we present the necessary background, including computational topology (Rips and Cech complexes, persistent homology...) and text mining (vector space model, similarity measure...). Then, we contrast our specialized approach with a previous approach using standard tools such as Rips complexes. We close by discussing connections with other fields as well as future research directions.

Joint work with Herbert Edelsbrunner.

Steve Y Oudot

Reflections in quiver and persistence theories

Reflections are among the classical operations used in quiver theory to transform a given quiver Q into another quiver Q′. The nice feature of such a reflection is to be associated with a functor mapping the representations of Q into representations of Q′. This is interesting in the context of the classification of quiver representations, where decompositions of representations of Q can be transferred to decompositions of representations of Q′. This principle has been applied with success by Bernstein, Gelfand and Ponomarev to prove Gabriel's theorem and some of its extensions in the 70's and 80's. In this talk I will draw a connection between reflection functors and the Diamond Principle from persistence theory, then I will present an application to the computation of persistence for zigzags.

Emmanuel Haucourt

Directions from Vector Fields

We wish to establish a relation between vector fields and certain directions.
Many of the d-spaces in nature are indeed carried by manifolds.
The directions of these d-spaces actually derive from vectors fields in a way that we will describe. On the way, we will introduce a special class of d-spaces that are better adapted to this approach : the complete (filled) d-spaces. In this framework, the parallelizable manifolds are of special interest. In particular they provide an important source of examples since every Lie group is parallellizable.

Damien Imbs

Untangling Partial Agreement: Iterated \$x\$-Consensus Simulations

In a basic model of distributed computing a set of asynchronous, crash prone processes communicate using a shared memory made of single-writer/multi-reader registers. This model is difficult to analyze. A more structured model is the Iterated Immediate Snapshot model (IIS), where processes execute communication closed rounds. In each round, they communicate using read/write objects that cannot be reused in later rounds. Yet, it is known that a task is solvable in the IIS model if and only if it is solvable in the usual read/write model. In fact, it is also known that both models are also equivalent when, in addition to read/write registers, processes also have access to slightly stronger communication objects called 01-tasks.

In this talk, I will extend the task computability equivalence further, and let processes use \$x\$-consensus objects. These objects allow solving consensus between up to \$x\$ processes. I will show that an iterated model where processes communicate through \$x\$-consensus objects is equivalent to the usual shared memory model augmented with \$x\$-consensus objects. In order to prove this result, I will present a simulation that, using only a sequence of \$x\$-consensus objects, allows executing a shared memory wait-free algorithm that uses \$x\$-consensus objects.

This result implies that the ability to invoke \$x\$-consensus in any order, instead of a fixed order, does not allow solving more tasks: the simulation ``untangles'' calls to \$x\$-consensus objects. Moreover, the simulation shows that \$x\$-consensus objects have a ``memory'' effect: these objects can implement any one-shot object that can be implemented using a shared memory.

Claudia Landi

Reducing Complexes in Multidimensional Persistent Homology

The Forman's discrete Morse theory appeared to be useful for providing filtration--preserving reductions of complexes in the study of persistent homology. So far, the algorithms computing discrete Morse matchings have only been used for one-dimensional filtrations. This talk deals with attempts in the direction of extending such algorithms to multidimensional filtrations.

Patrizio Frosini

Please look at the next page.

Author: Patrizio Frosini (University of Bologna)

Title: *Geometric shape comparison via G-invariant non-expansive operators and G-invariant persistent homology.*

Abstract. In many applications a geometric comparison of data with respect to a given invariance group G of homeomorphisms is required. Data are often represented by functions $f : X \to \mathbb{R}^k$ defined on a topological space X. This happens, e.g., in the case of pictures, where $X = \mathbb{R}^2$ and $k = 3$, with each color described by a vector in $\mathbb{R}^3$.

A way to compare this kind of data is given by the concept of *natural pseudo-distance* d_G. The pseudo-distance between $f_1 : X \to \mathbb{R}^k$ and $f_2 : X \to \mathbb{R}^k$ is defined as the infimum of the sup-distance between f_1 and the composition of f_2 with g, when g varies in the group G. If f_2 can be obtained from f_1 by composition with a transformation $g \in G$, the natural pseudo-distance between f_1 and f_2 vanishes. Roughly speaking, the natural pseudo-distance allows us to quantify to which extent two functions are similar to each other with respect to the invariance group G. Unfortunately, this pseudo-distance is not easy to compute.

In the last twenty-five years *persistent homology* has been developed as a mathematical tool for shape comparison in presence of noise, starting with the concept of *size function* (i.e. persistent homology in degree 0). In plain words, persistent homology in degree 0 describes the births and deaths of connected components in the sublevel sets of the considered function, while the level changes. Quite recently it has been proved that, in a suitable sense, persistent homology can be adapted to any invariance group G of homeomorphisms, and used to get easily computable lower bounds for the natural pseudo-distance d_G. This can be done by considering a collection of *G-invariant non-expansive operators*, applied to the data before computing their persistence diagrams.

In this talk we will briefly illustrate some theoretical results concerning this new approach to shape comparison. We will also show the preliminary results of an experiment concerning the comparison of some simple synthetic 2D grey-level images with respect to the group of isometries, produced by a forthcoming on-line demonstrator of the use of G-invariant persistent homology for image comparison (joint work with Grzegorz Jabłoński and Marc Ethier, Jagiellonian University, Kraków).

Petr Kuznetsov

A Generalized Asynchronous Computability Theorem

We consider the models of distributed computation defined as subsets of the runs of the iterated immediate snapshot model. We show that the class of models is expressive enough to capture generic share-memory systems that describe non-uniform and/or correlated failures. Given a task T and a model M, we then provide topological conditions for T to be solvable in M. When applied to the classical wait-free model, including all possible runs, our conditions result in the celebrated Asynchronous Computability Theorem (ACT) of Herlihy and Shavit. More generally, our results imply a topological characterization of task solvability in any "adversarial" shared-memory model.

Thomas Nowak

Point-Set Topology for Impossibility Results in Distributed Computing

This talk provides an elementary introduction to the point-set topology of exeuction spaces and provides applications to solvability questions in distributed computing. We focus on the binary consensus problem, for which the decision function is continuous and hence cannot map a connected space to a disconnected one. Now, the space of possible decisions for binary consensus is discrete with two elements, and the introduction of certain faults ensures a connected execution space. This then creates a contradiction to solvability of binary consensus. We exemplify the approach by re-proving three classical impossibility results.

Samuel Mimram

Dihomotopy and the cube property

Concurrent programs can be modeled as topological spaces which are directed, i.e. equipped with a notion of time direction. In such a model, a directed path corresponds to an execution of the program, and a directed homotopy between directed paths to an equivalence wrt to permutation of independent actions in schedulings of the program. One of the original motivations for this point of view was the hope that one could use of classical methods in algebraic topology in order to provide invariants of programs, such as homology groups. In most cases, the classical tools need to be reworked heavily in order to be adapted to the directed case. However, I will show here that, for semantics of programs using mutexes only, dihomotopy coincides with homotopy thus allowing for more direct computations. This comes from the fact that these semantics are geometric realizations of particular precubical sets, satisfying a "cube axiom", and I will present an algebraic proof in this setting. Interestingly, the resulting spaces are non-positively curved and thus enjoy other properties that I will present. This is joint work with Éric Goubault.

Thomas Kahl

Reduction of higher-dimensional automata

Higher-dimensional automata constitute one of the most expressive models for concurrent systems. An important practical problem in concurrency theory is the fact that models of systems can easily become very large. In this talk, I will discuss "topological abstraction" of higher-dimensional automata. This means the replacement of an HDA by a smaller one that models the same system and can be considered weakly equivalent from the point of view of directed algebraic topology. I will present some results that permit one to reduce the size of an HDA through cube merging and collapsing operations.