Geometric shape comparison via G-invariant non-expansive operators and G-invariant persistent homology

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Outline

Our problem

Mathematical setting and theoretical results

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An example in shape comparison

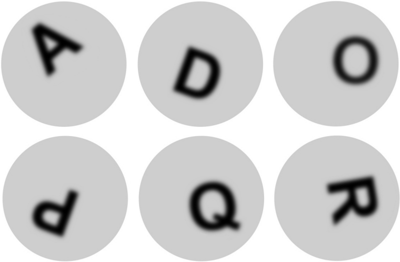


Figure: Examples of letters $\mathtt{A}, \mathtt{D}, \mathtt{O}, \mathtt{P}, \mathtt{Q}, \mathtt{R}$ represented by functions $\varphi_{\mathtt{A}}, \varphi_{\mathtt{D}}, \varphi_{\mathtt{O}}, \varphi_{\mathtt{P}}, \varphi_{\mathtt{Q}}, \varphi_{\mathtt{R}}$ from $\mathbb{R}^2$ to the real numbers. Each function $\varphi_Y : \mathbb{R}^2 \to \mathbb{R}$ describes the grey level at each point of the topological space $\mathbb{R}^2$, with reference to the considered instance of the letter Y. Black and white correspond to the values 0 and 1, respectively (so that light grey corresponds to a value close to 1).

A letter O

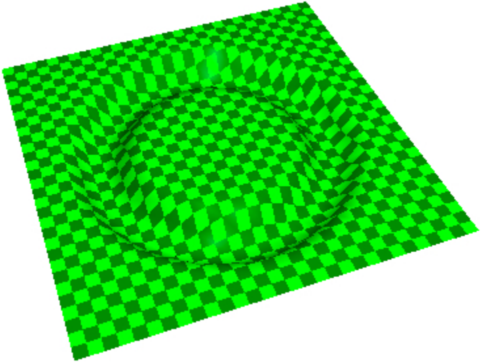


Figure: Part of the graph of a function representing a letter O.

Key observation

Persistent homology is invariant with respect to ANY homeomorphism!

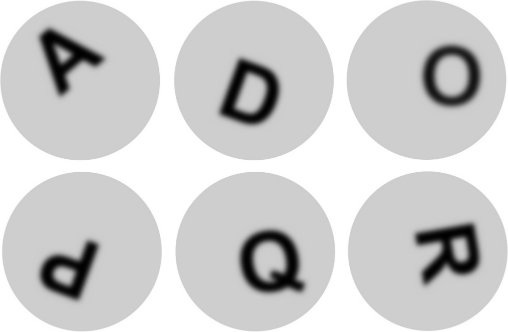
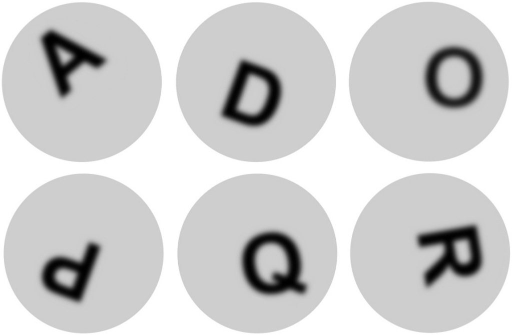


Figure: These functions share the same persistent homology.

Main question

How can we use persistent homology to distinguish these letters?



We have to restrict the invariance of persistent homology.

Couldn't we maintain classical persistent homology?

One could think of using other filtering functions, possibly defined on different topological spaces. For example, we could extract boundaries of letters and consider the distance from the center of mass of each boundary. This approach presents some drawbacks:

1. It "forgets" most of the information contained in the image $\varphi : \mathbb{R}^2 \to \mathbb{R}$ that we are considering, confining itself to examine the boundary of the letter represented by φ.
2. It usually requires an extra computational cost (e.g., to extract the boundaries of the letters).
3. It can produce a different topological space for each new filtering function (e.g., this happens for letters).
4. ABOVE ALL: It is not clear how we can translate the invariance that we need into the choice of new filtering functions defined on new topological spaces.

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Natural pseudo-distance associated with a group G

Definition

Let X be a compact space. Let G be a subgroup of the group $\mathrm{Homeo}(X)$ of all homeomorphisms $f : X \to X$. The pseudo-distance $d_G : C^0(X,\mathbb{R}) \times C^0(X,\mathbb{R}) \to \mathbb{R}$ defined by setting

$$d_G(\varphi,\psi) = \inf_{g\in G} \max_{x\in X} |\varphi(x) - \psi(g(x))|$$

is called the natural pseudo-distance associated with the group G.

In plain words, the definition of d_G is based on the attempt of finding the best correspondence between the functions φ, ψ by means of homeomorphisms in G.

G-invariant non-expansive operators

The natural pseudo-distance d_G represents our ground truth.

Unfortunately, d_G is difficult to compute. This is also a consequence of the fact that we can easily find topological subgroups G of $\mathrm{Homeo}(X)$ that cannot be approximated with arbitrary precision by smaller finite subgroups of G (i.e. $G =$ group of rigid motions of $X = \mathbb{R}^3$).

In this talk we will show that d_G can be approximated with arbitrary precision by means of a DUAL approach based on persistent homology and G-invariant non-expansive operators.

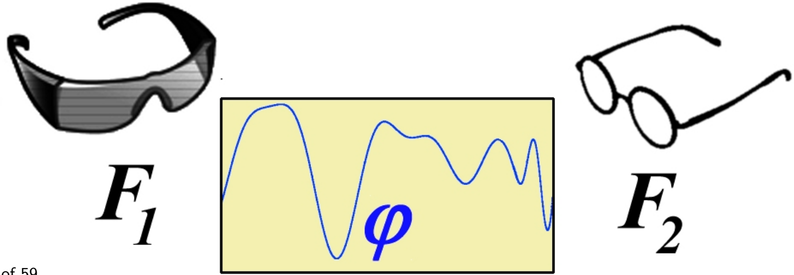
Research based on an ongoing joint research project with

Grzegorz Jabłoński and Marc Ethier
Jagiellonian University - Kraków

G-invariant non-expansive operators

Informal description of our idea

Instead of changing the topological space X, we can get invariance with respect to the group G by changing the "glasses" that we use "to observe" the filtering functions. In our approach, these "glasses" are G-operators F_i, which act on the filtering functions.



G-invariant non-expansive operators

Let us consider the following objects:

- A triangulable space X with nontrivial homology in degree k.
- A set Φ of continuous functions from X to $\mathbb{R}$, that contains the set of all constant functions.
- A topological subgroup G of $\mathrm{Homeo}(X)$ that acts on Φ by composition on the right.
- The natural pseudo-distance d_G on Φ with respect to G, defined by setting $d_G(\varphi_1, \varphi_2) := \inf_{g \in G} \|\varphi_1 - \varphi_2 \circ g\|_\infty$ for every $\varphi_1, \varphi_2 \in \Phi$.
- The distance d_∞ on Φ, defined by setting $d_\infty(\varphi_1, \varphi_2) := \|\varphi_1 - \varphi_2\|_\infty$. This is just the natural pseudo-distance d_G in the case that G is the trivial group $\mathbf{I} = \{id\}$, containing only the identical homeomorphism.
- A subset $\mathscr{F}$ of the set $\mathscr{F}^{\mathrm{all}}(\Phi, G)$ of all non-expansive G-operators from Φ to Φ.

The operator space $\mathscr{F}^{\mathrm{all}}(\Phi, G)$

In plain words, $F \in \mathscr{F}^{\mathrm{all}}(\Phi, G)$ means that

1. $F : \Phi \longrightarrow \Phi$
2. $F(\varphi \circ g) = F(\varphi) \circ g$. ($F$ is a G-operator)
3. $\|F(\varphi_1) - F(\varphi_2)\|_\infty \leq \|\varphi_1 - \varphi_2\|_\infty$. ($F$ is non-expansive)

The operator F is not required to be linear.

Some simple examples of F, taking Φ equal to the set of all continuous functions $\varphi : \mathbf{S}^1 \to \mathbb{R}$ and G equal to the group of all rotations of $\mathbf{S}^1$:

- $F(\varphi) :=$ the constant function $\psi : \mathbf{S}^1 \to \mathbb{R}$ taking the value $\max \varphi$;
- $F(\varphi)$ defined by setting $F(\varphi)(x) := \max\left\{\varphi\left(x - \frac{\pi}{8}\right), \varphi\left(x + \frac{\pi}{8}\right)\right\}$;
- $F(\varphi)$ defined by setting $F(\varphi)(x) := \frac{1}{2}\left(\varphi\left(x - \frac{\pi}{8}\right) + \varphi\left(x + \frac{\pi}{8}\right)\right)$.

The pseudo-metric $D^{\mathscr{F}}_{\mathrm{match}}$

For every $\varphi_1, \varphi_2 \in \Phi$ we set

$$D^{\mathscr{F}}_{\mathrm{match}}(\varphi_1, \varphi_2) := \sup_{F \in \mathscr{F}} d_{match}(\rho_k(F(\varphi_1)), \rho_k(F(\varphi_2)))$$

where $\rho_k(\psi)$ denotes the persistent Betti number function (i.e. the rank invariant) of ψ in degree k.

Proposition

$D^{\mathscr{F}}_{match}$ is a G-invariant and stable pseudo-metric on Φ.

The G-invariance of $D^{\mathscr{F}}_{match}$ means that $D^{\mathscr{F}}_{\mathrm{match}}(\varphi_1, \varphi_2 \circ g) = D^{\mathscr{F}}_{\mathrm{match}}(\varphi_1, \varphi_2)$ for every $\varphi_1, \varphi_2 \in \Phi$ and every $g \in G$.

An equivalence result

We observe that the pseudo-distance $D^{\mathscr{F}}_{\text{match}}$ and the natural pseudo-distance d_G are defined in quite different ways.

In particular, the definition of $D^{\mathscr{F}}_{\text{match}}$ is based on persistent homology, while the natural pseudo-distance d_G is based on the group of homeomorphisms G.

In spite of this, the following statement holds:

Theorem

If $\mathscr{F} = \mathscr{F}^{\text{all}}(\Phi, G)$, then the pseudo-distance $D^{\mathscr{F}}_{\text{match}}$ coincides with the natural pseudo-distance d_G on Φ.

Our main idea

The previous theorem suggests to study $D^{\mathscr{F}}_{\mathrm{match}}$ instead of d_G.

To this end, let us choose a finite subset $\mathscr{F}^*$ of $\mathscr{F}$, and consider the pseudo-metric

$$D^{\mathscr{F}^*}_{\mathrm{match}}(\varphi_1, \varphi_2) := \max_{F \in \mathscr{F}^*} d_{match}(\rho_k(F(\varphi_1)), \rho_k(F(\varphi_2)))$$

for every $\varphi_1, \varphi_2 \in \Phi$.

Obviously, $D^{\mathscr{F}^*}_{\mathrm{match}} \leq D^{\mathscr{F}}_{\mathrm{match}}$.

Furthermore, if $\mathscr{F}^*$ is dense enough in $\mathscr{F}$, then the new pseudo-distance $D^{\mathscr{F}^*}_{\mathrm{match}}$ is close to $D^{\mathscr{F}}_{\mathrm{match}}$.

In order to make this point clear, we need the next theoretical result.

Compactness of $\mathscr{F}^{\mathrm{all}}(\Phi, G)$

The following result holds:

Theorem

If (Φ, d_∞) is a compact metric space, then $\mathscr{F}^{\mathrm{all}}(\Phi, G)$ is a compact metric space with respect to the distance d defined by setting

$$d(F_1, F_2) := \max_{\varphi \in \Phi} \| F_1(\varphi) - F_2(\varphi) \|_\infty$$

for every $F_1, F_2 \in \mathscr{F}$.

Approximation of $\mathscr{F}^{\mathrm{all}}(\Phi, G)$

This statement follows:

Corollary

Assume that the metric space (Φ, d_∞) is compact. Let $\mathscr{F}$ be a subset of $\mathscr{F}^{\mathrm{all}}(\Phi, G)$. For every $\varepsilon > 0$, a finite subset $\mathscr{F}^*$ of $\mathscr{F}$ exists, such that

$$\left| D_{match}^{\mathscr{F}^*}(\varphi_1, \varphi_2) - D_{match}^{\mathscr{F}}(\varphi_1, \varphi_2) \right| \le \varepsilon$$

for every $\varphi_1, \varphi_2 \in \Phi$.

This corollary implies that the pseudo-distance $D_{match}^{\mathscr{F}}$ can be approximated computationally, at least in the compact case.

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Let us check what happens in practice

A RETRIEVAL EXPERIMENT
ON A DATASET OF CURVES

Let us check what happens in practice

We have considered

1. a dataset of 10000 functions from $\mathbf{S}^1$ to $\mathbb{R}$, depending on five random parameters (#);
2. these three invariance groups:
   - the group $\mathrm{Homeo}(\mathbf{S}^1)$ of all self-homeomorphisms of $\mathbf{S}^1$;
   - the group $R(\mathbf{S}^1)$ of all rotations of $\mathbf{S}^1$;
   - the trivial group $\mathbf{I}(\mathbf{S}^1) = \{id\}$, containing just the identity of $\mathbf{S}^1$.

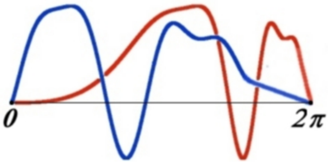
Obviously,

$$\mathrm{Homeo}(\mathbf{S}^1) \supset R(\mathbf{S}^1) \supset \mathbf{I}(\mathbf{S}^1).$$

(#) For $1 \leq i \leq 10000$ we have set $\bar{\varphi}_i(x) = r_1 \sin(3x) + r_2 \cos(3x) + r_3 \sin(4x) + r_4 \cos(4x)$, with $r_1, .., r_4$ randomly chosen in the interval $[-2, 2]$; the i-th function in our dataset is the function $\varphi_i := \bar{\varphi}_i \circ \gamma_i$, where $\gamma_i(x) := 2\pi(\frac{x}{2\pi})^{r_5}$ and r_5 is randomly chosen in the interval $[\frac{1}{2}, 2]$.

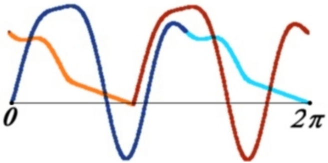
Let us check what happens in practice

The choice of $\mathrm{Homeo}(\mathbf{S}^1)$ as an invariance group implies that the following two functions are considered equivalent. Their graphs are obtained from each other by applying a horizontal stretching. Also shifts are accepted as legitimate transformations.



Let us check what happens in practice

The choice of $R(\mathbf{S}^1)$ as an invariance group implies that the following two functions are considered equivalent. Their graphs are obtained from each other by applying a rotation of $\mathbf{S}^1$. Stretching is not accepted as a legitimate transformation.



Finally, the choice of $\mathbf{I}(\mathbf{S}^1) = \{id\}$ as an invariance group means that two functions are considered equivalent if and only if they coincide everywhere.

The results of an experiment: the group $\mathrm{Homeo}(\mathbf{S}^1)$

What happens if we decide to assume

that the invariance group is the group $\mathrm{Homeo}(\mathbf{S}^1)$

of all self-homeomorphisms of $\mathbf{S}^1$?

The results of an experiment: the group $\mathrm{Homeo}(\mathbf{S}^1)$

If we choose $G = \mathrm{Homeo}(\mathbf{S}^1)$, to proceed we need to choose a finite set of non-expansive $\mathrm{Homeo}(\mathbf{S}^1)$-operators. In our experiment we have considered these three non-expansive $\mathrm{Homeo}(\mathbf{S}^1)$-operators:

- $F_0 := id$ (i.e., $F_0(\varphi) := \varphi$);
- $F_1 := -id$ (i.e., $F_0(\varphi) := -\varphi$);
- $F_2(\varphi) :=$ the constant function $\psi : \mathbf{S}^1 \to \mathbb{R}$ taking the value $\frac{1}{5} \cdot \sup\{-\varphi(x_1) + \varphi(x_2) - \frac{1}{2}\varphi(x_3) + \frac{1}{2}\varphi(x_4) - \varphi(x_5) + \varphi(x_6)\}$, $(x_1, \ldots, x_6)$ varying among all the counterclockwise 6-tuples on $\mathbf{S}^1$.

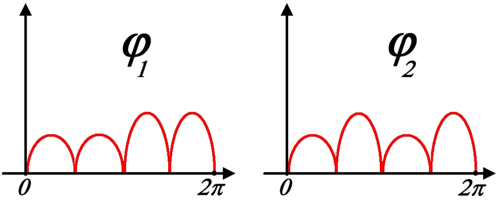
This choice produces the $\mathrm{Homeo}(\mathbf{S}^1)$-invariant pseudo-distance

$$D_{match}^{\mathscr{F}^*}(\varphi_1, \varphi_2) := \max_{0 \le i \le 2} d_{match}(\rho_k(F_i(\varphi_1)), \rho_k(F_i(\varphi_2))).$$

An important remark

It is important to use several operators. The use of just one operator still produces a pseudo-distance $D^{\mathscr{F}^*}_{match}$ that is invariant under the action of the group G, but this choice is far from guaranteeing a good approximation of the natural pseudo-distance d_G.

As an example in the case $G = \mathrm{Homeo}(\mathbf{S}^1)$, if we use just the identity operator (i.e., we just apply classical persistent homology), we cannot distinguish these two functions $\varphi_1, \varphi_2 : \mathbf{S}^1 \to \mathbb{R}$, despite the fact that they are different for d_G:



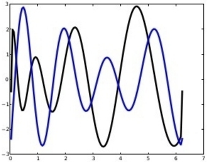
The results of an experiment: the group Homeo($\mathbf{S}^1$)

Here is a query (in **blue**), and the first four retrieved functions (in **black**):

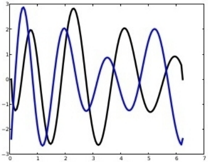
$D^{\mathcal{F}^*}_{\text{match}}(\varphi_1, \varphi_2)$

Mean	Max
1.648	4.831

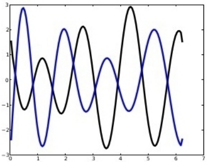
Standard deviation
0.934



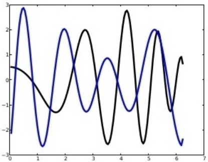
(a) φ_{516}, dist: 0.0465393



(b) φ_{381}, dist: 0.0541687



(c) φ_{7776}, dist: 0.0984192

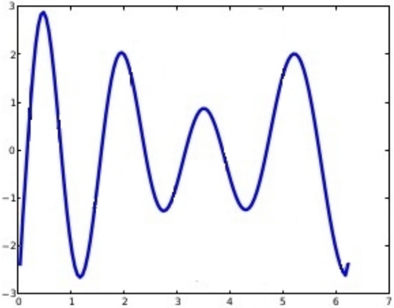


(d) φ_{6214}, dist: 0.10376

The results of an experiment: the group $\mathrm{Homeo}(\mathbf{S}^1)$

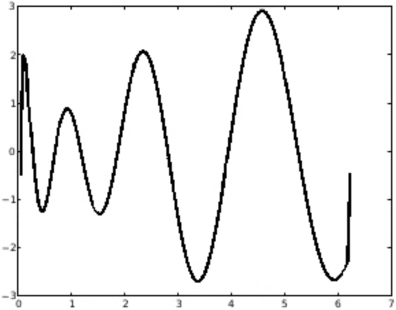
Let's have a closer look at the query and at the first retrieved function:

Here is the query:



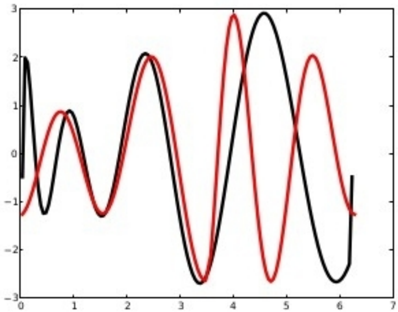
The results of an experiment: the group $\mathrm{Homeo}(\mathbf{S}^1)$

Here is the first retrieved function with respect to $D_{match}^{\mathscr{F}^*}$:



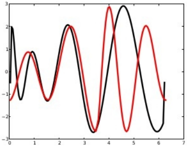
The results of an experiment: the group $\mathrm{Homeo}(\mathbf{S}^1)$

Here is the query function after aligning it to the first retrieved function by means of a shift (in **red**).
The first retrieved function is represented in **black**.
The figure shows that the retrieved function is approximately equivalent to the query function, by applying a shift and a stretching.



The results of an experiment: the group Homeo($\mathbf{S}^1$)

Here is the query function after aligning it to the first four retrieved functions by means of a shift (in **red**).
The first four retrieved functions are represented in **black**.



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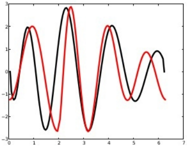
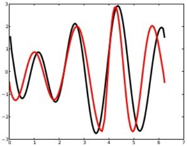
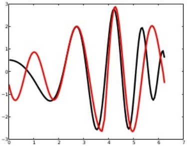


(b) φ_{381}, dist: 0.0541687



(c) φ_{7776}, dist: 0.0984192



(d) φ_{6214}, dist: 0.10376

The results of an experiment: the group $R(\mathbf{S}^1)$

What happens if we decide to assume

that the invariance group is the group $R(\mathbf{S}^1)$

of all rotations of $\mathbf{S}^1$?

The results of an experiment: the group $R(\mathbf{S}^1)$

If we choose $G = R(\mathbf{S}^1)$, in order to proceed we need to choose a finite set of non-expansive $R(\mathbf{S}^1)$-operators. Obviously, since F_0, F_1 and F_2 are $\mathrm{Homeo}(\mathbf{S}^1)$-invariant, they are also $R(\mathbf{S}^1)$-invariant. In our experiment we have added these five non-expansive $R(\mathbf{S}^1)$-operators (which are <u>not</u> $\mathrm{Homeo}(\mathbf{S}^1)$-invariant) to F_0, F_1 and F_2:

- $F_3(\varphi)(x) := \max\{\varphi(x), \varphi(x+\pi)\}$
- $F_4(\varphi)(x) := \frac{1}{2}\cdot\left(\varphi(x)+\varphi(x+\frac{\pi}{4})\right)$
- $F_5(\varphi)(x) := \max\{\varphi(x), \varphi(x+\pi/10), \varphi(x+\frac{2\pi}{10}), \varphi(x+\frac{3\pi}{10})\}$
- $F_6(\varphi)(x) := \frac{1}{3}\cdot\left(\varphi(x)+\varphi(x+\frac{\pi}{3})+\varphi(x+\frac{\pi}{4})\right)$
- $F_7(\varphi)(x) := \frac{1}{3}\cdot\left(\varphi(x)+\varphi(x+\frac{\pi}{3})+\varphi(x+\frac{2\pi}{3})\right)$

This choice produces the $R(\mathbf{S}^1)$-invariant pseudo-distance

$$D^{\mathscr{F}^*}_{match}(\varphi_1,\varphi_2) := \max_{0\le i\le 7} d_{match}(\rho_k(F_i(\varphi_1)), \rho_k(F_i(\varphi_2))).$$

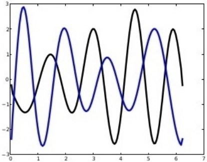
The results of an experiment: the group $R(\mathbf{S}^1)$

Here is a query (in **blue**), and the first four retrieved functions (in **black**):

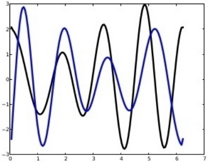
$D^{\mathcal{F}^*}_{\text{match}}(\varphi_1, \varphi_2)$

Mean	Max
1.938	4.831

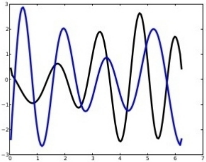
Standard deviation
0.874



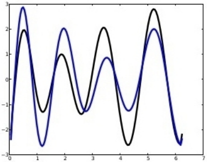
(a) φ_{5566}, dist: 0.333405



(b) φ_{8454}, dist: 0.422668



(c) φ_{8909}, dist: 0.453949

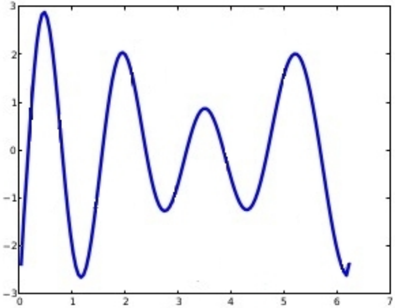


(d) φ_{4426}, dist: 0.46463

The results of an experiment: the group $R(\mathbf{S}^1)$

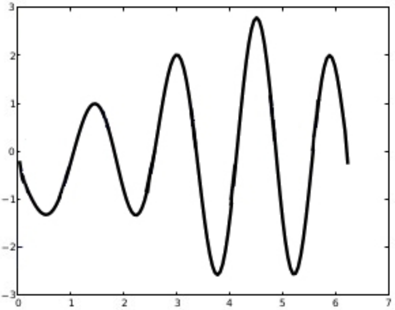
Let's have a closer look at the query and at the first retrieved function:

Here is the query:



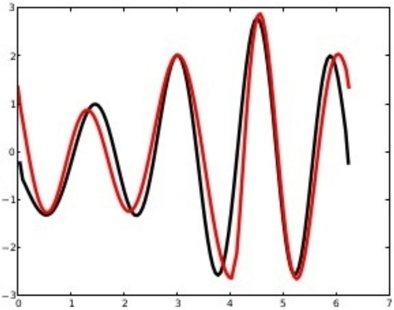
The results of an experiment: the group $R(\mathbf{S}^1)$

Here is the first retrieved function with respect to $D^{\mathscr{F}^*}_{match}$:



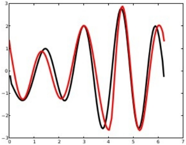
The results of an experiment: the group $R(\mathbf{S}^1)$

Here is the query function after aligning it to the first retrieved function by means of a shift (in **red**).
The first retrieved function is represented in **black**.
The figure shows that the retrieved function is approximately equivalent to the query function, via a shift.

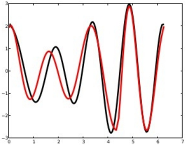


The results of an experiment: the group $R(\mathbf{S}^1)$

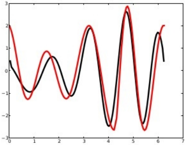
Here is the query function after aligning it to the first four retrieved functions by means of a shift (in **red**).
The first four retrieved functions are represented in **black**.



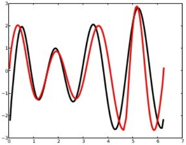
(a) φ_{5566}, dist: 0.333405



(b) φ_{8454}, dist: 0.422668



(c) φ_{8909}, dist: 0.453949



(d) φ_{4426}, dist: 0.46463

The results of an experiment: the group $\mathbf{I}(\mathbf{S}^1)$

Finally, what happens if we decide to assume

that the invariance group is the group $\mathbf{I}(\mathbf{S}^1) = \{id\}$

containing only the identity of $\mathbf{S}^1$?

This means that the “perfect” retrieved function

should coincide with our query.

The results of an experiment: the group $\mathbf{I}(\mathbf{S}^1)$

If we choose $G = \mathbf{I}(\mathbf{S}^1) = \{id\}$, in order to proceed we need to choose a finite set of non-expansive operators (obviously, every operator is an $\mathbf{I}(\mathbf{S}^1)$-operator).

In our experiment we have considered these three non-expansive operators (which are not $R(\mathbf{S}^1)$-operators):

- $F_8(\varphi)(x) := \sin(x)\varphi(x)$
- $F_9(\varphi)(x) := \frac{\sqrt{2}}{2}\sin(x)\varphi(x) + \frac{\sqrt{2}}{2}\cos(x)\varphi(x+\frac{\pi}{2})$
- $F_{10}(\varphi)(x) := \sin(2x)\varphi(x)$

We have added F_8, F_9, F_{10} to $F_1, \ldots, F_7$.

This choice produces the pseudo-distance

$$D_{match}^{\mathscr{F}^*}(\varphi_1, \varphi_2) := \max_{0 \le i \le 10} d_{match}(\rho_k(F_i(\varphi_1)), \rho_k(F_i(\varphi_2))).$$

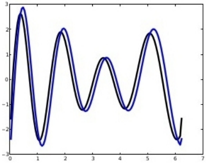
The results of an experiment: the group $\mathbf{I}(\mathbf{S}^1)$

Here is a query (in **blue**), and the first four retrieved functions (in **black**):

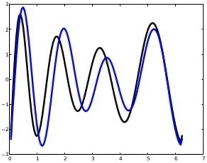
$$D^{\mathcal{F}^*}_{\text{match}}(\varphi_1, \varphi_2)$$

Mean	Max
2.022	4.831

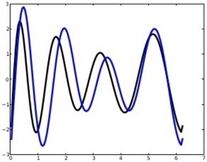
Standard deviation
0.828



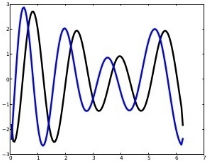
(a) φ_{7133}, dist: 0.415802



(b) φ_{7001}, dist: 0.598145



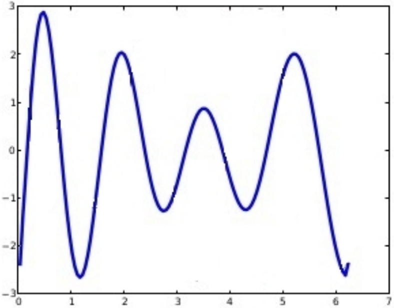
(c) φ_{389}, dist: 0.617218



(d) φ_{5723}, dist: 0.617981

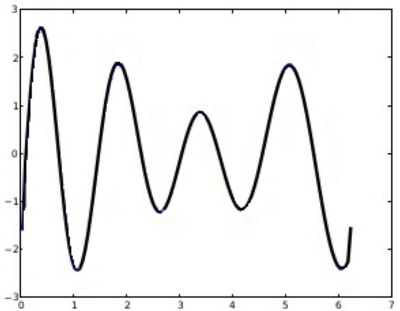
The results of an experiment: the group $\mathbf{I}(\mathbf{S}^1)$

Let's have a closer look at the query and at the first retrieved function:
Here is the query:



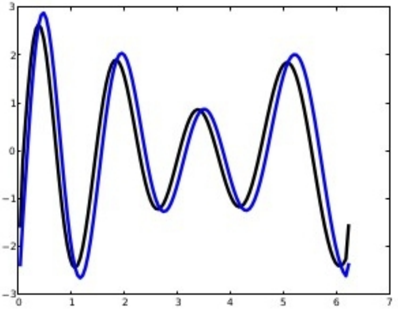
The results of an experiment: the group $\mathbf{I}(\mathbf{S}^1)$

Here is the first retrieved function with respect to $D^{\mathscr{F}}_{match}$:



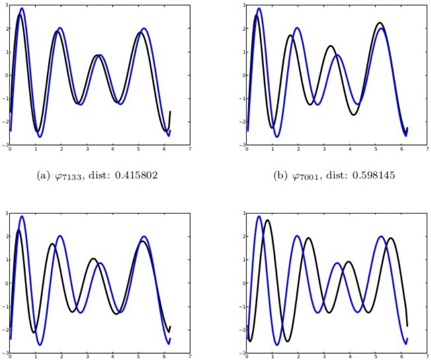
The results of an experiment: the group $\mathbf{I}(\mathbf{S}^1)$

The first retrieved function is represented in **black**.
As expected, no aligning shift is necessary here.
The figure shows that the retrieved function is approximately equal to the query function.



The results of an experiment: the group $\mathbf{I}(\mathbf{S}^1)$

Here we show again the query function and the first four retrieved functions (in **black**).
The figure shows that the retrieved functions are approximately coinciding with the query function.



(a) φ_{7133}, dist: 0.415802

(b) φ_{7001}, dist: 0.598145

An open problem

We have proven that if Φ is compact, then $D^{\mathscr{F}}_{match}$ can be approximated computationally.

However, this result does not say which set of operators allows for both a good approximation of $D^{\mathscr{F}}_{match}$ and a fast computation.

Further research is needed in this direction.

Our problem

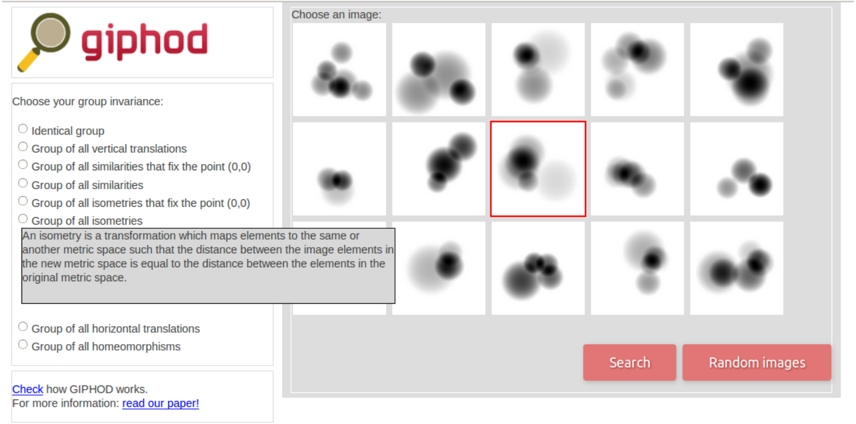
Mathematical setting and theoretical results

Experiments

GIPHOD

GIPHOD

GIPHOD: joint project with Grzegorz Jabłoński and Marc Ethier (Jagiellonian University - Kraków)



GIPHOD (Group Invariant Persistent Homology On-line Demonstrator)

GIPHOD is an on-line demonstrator, allowing the user to choose an image and an invariance group. GIPHOD searches for the most similar images in the dataset, with respect to the chosen invariance group.

Purpose: to show the use of G-invariant persistent homology for image comparison.

Dataset: 10.000 grey-level synthetic images obtained by adding randomly chosen bell-shaped functions.

GIPHOD SHOULD BE AVAILABLE IN THE NEXT FEW MONTHS.

GIPHOD (Group Invariant Persistent Homology On-line Demonstrator)

We are going to show the results of an experiment where the invariance group G is the group of isometries:

Some data about the pseudo-metric $D^{\mathscr{F}}_{match}$ in this case:

- The images are coded as functions from $\mathbb{R}^2 \to [0,1]$;
- Mean distance between images: 0.35752;
- Standard deviation of distance between images: 0.14881;
- Number of GINOs that have been used: 12.

GIPHOD (Group Invariant Persistent Homology On-line Demonstrator)

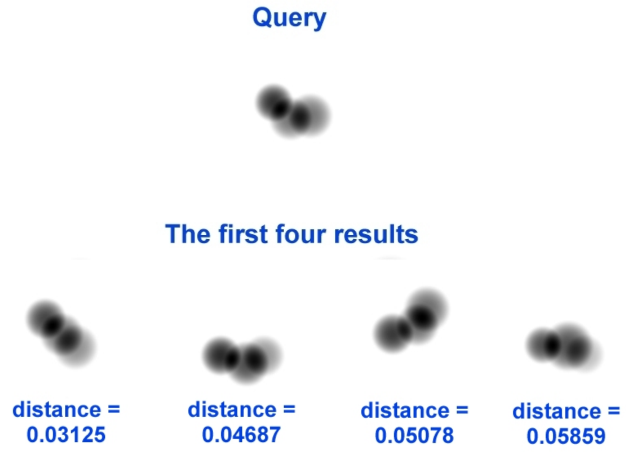
List of GINOs that have been used in the following image retrievals, where the invariance group G is the group of isometries:

- $F(\varphi) = \varphi$.
- $F(\varphi) :=$ constant function taking each point to the value $\int_{\mathbb{R}^2} \varphi(\mathbf{x})\ d\mathbf{x}$.
- $F(\varphi)$ defined by setting

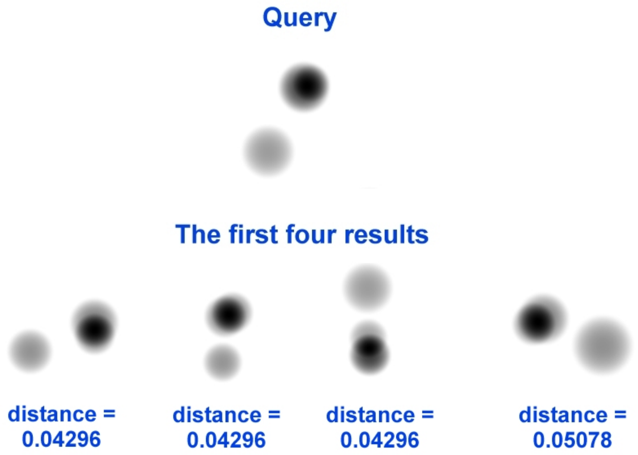
$$F(\varphi)(\mathbf{x}) := \int_{\mathbb{R}^2} \varphi(\mathbf{x} - \mathbf{y}) \cdot \beta\left(\|\mathbf{y}\|_2\right)\ d\mathbf{y}$$

  where $\beta : \mathbb{R} \to \mathbb{R}$ is an integrable function with $\int_{\mathbb{R}^2} \left|\beta\left(\|\mathbf{y}\|_2\right)\right|\ d\mathbf{y} \leq 1$. Four GINOs of this kind have been used.
- The opposite operators $-F$ of the six previous GINOs.

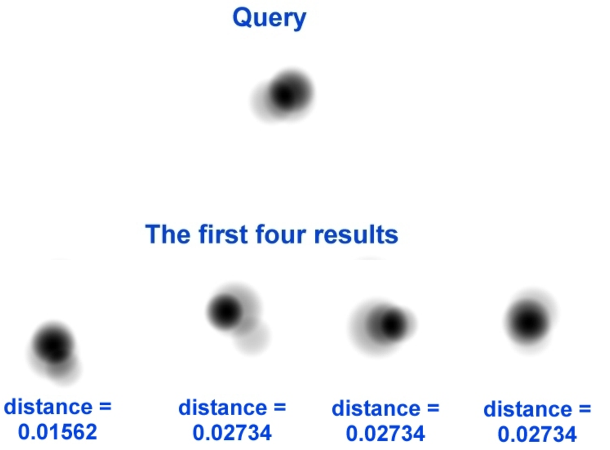
GIPHOD: Examples for the group of isometries



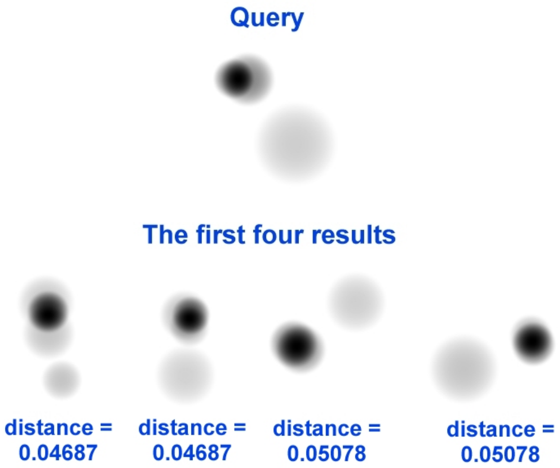
GIPHOD: Examples for the group of isometries



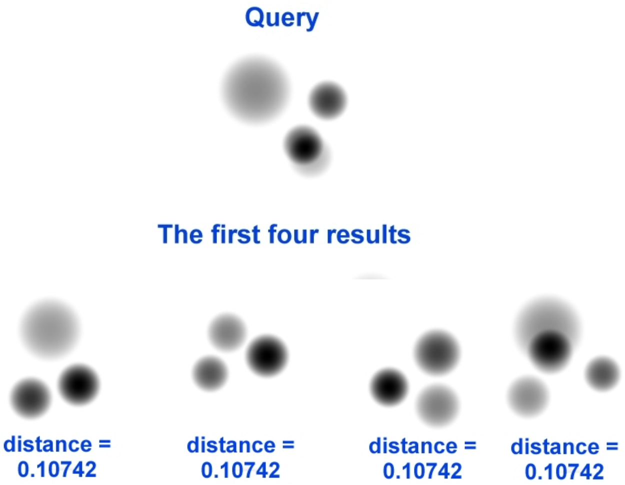
GIPHOD: Examples for the group of isometries



GIPHOD: Examples for the group of isometries



GIPHOD: Examples for the group of isometries



Conclusions

In this talk we have shown that

- Persistent homology can be adapted to proper subgroups of the group of all self-homeomorphisms of a triangulable space X, in order to approximate the natural pseudo-metric d_G. This can be done by means of a method that is based on non-expansive G-operators and can be used for any subgroup G of $\mathrm{Homeo}(X)$. This method is stable with respect to noise.
- Some theoretical results and two experiments concerning this method have been illustrated, showing the possible use of this approach for data retrieval.

For more information about the approach described in these slides click on the following link: http://arxiv.org/pdf/1312.7219v3.pdf.

THANKS FOR YOUR ATTENTION!