

**Solutions to exercise Kreyszig 6.2.11**

$$y'' + ay' + by = r(t) \quad ; \quad y(0) = 1; y'(0) = 31^{1/2}$$

$$a = 3; b = 9/4$$

$$r(t) = 9t^3 + 64 \rightarrow \text{by table} \rightarrow R(s) = 54/s^4 + 64/s$$

General form of the solution is:

$$(s^2 + as + b)Y(s) = (s + a)y(0) + y'(0) + R(s)$$

So in this case:

$$(s^2 + 3s + 9/4)Y(s) = (s + 3)1 + 31^{1/2} + R(s)$$

$$(s + 3/2)^2 Y(s) = (s + 3/2) + \frac{33s^4 + 64s^3 + 54}{s^4}$$

$$Y(s) = \frac{1}{(s + 3/2)} + \frac{33s^4 + 64s^3 + 54}{(s + 3/2)^2 s^4}$$

By partial fraction decomposition and solving, this becomes (see \*):

$$Y(s) = \frac{1}{(s + 3/2)} + \frac{A}{s^4} + \frac{B}{s^3} + \frac{C}{s^2} + \frac{D}{s} + \frac{E}{(s + 3/2)^2} + \frac{F}{(s + 3/2)}$$

$$\text{with } A = 24; B = -32; C = 32; D = 0; E = 1; F = 0$$

So:

$$Y(s) = \frac{1}{(s + 3/2)} + \frac{24}{s^4} + \frac{-32}{s^3} + \frac{32}{s^2} + \frac{1}{(s + 3/2)^2}$$

By the table and the first shifting theorem it follows that:

$$y(t) = e^{-3/2t} + 4t^3 - 16t^2 + 32t + te^{-3/2t}$$

\*Partial fraction decomposition by:

$$\begin{aligned} \frac{A}{s^4} + \frac{B}{s^3} + \frac{C}{s^2} + \frac{D}{s} + \frac{E}{(s + 3/2)^2} + \frac{F}{(s + 3/2)} &= \frac{33s^4 + 64s^3 + 54}{(s + 3/2)^2 s^4} \\ A(s^2 + 3s + 9/4) + B(s^3 + 3s^2 + 9/4s) + C(s^4 + 3s^3 + 9/4s^2) + \\ + D(s^5 + 3s^4 + 9/4s^3) + E(s^4) + F(s^5 + 9/4s^4) &= 33s^4 + 64s^3 + 54 \end{aligned}$$

Ordering terms by power of  $s$ :

$$\begin{array}{ll} s^0 : 9/4 = 54 \Rightarrow & A = 24 \\ s^1 : 3A + 9/4B = 0 \Rightarrow & B = -32 \\ s^2 : A + 3B + 9/4C = 0 \Rightarrow & C = 32 \\ s^3 : B + 3C + 9/4D = 64 \Rightarrow & D = 0 \\ s^4 : C + 3D + E + 3/2F = 33 \Rightarrow & E = 1 \\ s^5 : D + F = 0 \Rightarrow & F = 0 \rightarrow (\text{put in } s^4 \text{ equation}) \end{array}$$