Author: J.O. Hooghoudt ; email:janotto@math.aau.dk ; date=02/10/13

Solution to exercize Kreyszig 6.7.19

By following e.g. example 2 of section 6.7 we find (by Kirchhoffs Voltage Law) for this network the following equations

$$4i_1 + (i_1 - i_2)8 + 2i_1^{'} = v(t)$$
$$8i_2 + 4i_2^{'} + 8(i_2 - i_1) = 0$$

with $v(t) = 390\cos(t)$. Laplace transformed these equations become (as $i_1(0) = i_2(0) = 0$):

$$4I_1 + (I_1 - I_2)8 + 2sI_1 = 390\frac{s}{s^2+1}$$
$$8I_2 + 4sI_2 + 8(I_2 - I_1) = 0$$

So

$$(2s+12)I_1 - 8I_2 = 390\frac{s}{s^2+1} \qquad (i)$$
$$(4s+16)I_2 - 8I_1 = 0 \Rightarrow I_1 = (s/2+2)I_2 \qquad (ii)$$

Then from (i):

$$(2s+12)(s/2+2)I_2 - 8I_2 = 390\frac{s}{s^2+1}$$
$$(s^2+10s+16)I_2 = 390\frac{s}{s^2+1}$$
$$I_2 = 390\frac{s}{(s^2+1)(s+8)(s+2)}$$

By partial fraction decompostion(see $*$)

$$I_2 = \frac{18s}{s^2+1} + \frac{12}{s^2+1} + \frac{8}{s+8} + \frac{-26}{s+2}$$

and by the aid of the table or your brain/memory :-)

$$i_2(t) = 18\cos(t) + 12\sin(t) + 8e^{-8t} - 26e^{-2t}$$

$*$Partial fraction decomposition by:

$$\frac{As+B}{s^2+1} + \frac{C}{s+8} + \frac{D}{s+2} = 390\frac{s}{(s^2+1)(s+8)(s+2)}$$

So:

$$(As + B)(s^2 + 10s + 16) + C(s^3 + s + 2s^2 + 2) + D(s^3 + s + 8s^2 + 8) = 390s$$

Ordering terms by power of s:

$$\begin{aligned}
s^0 : 16B + 2C + 8D &= 0 \Rightarrow B = (-C - 4D)/8 \\
s^1 : 16A + 10B + C + D &= 390 \\
s^2 : 10A + B + 2C + 8D &= 0 \Rightarrow 10(-C - D) + (-C - 4D)/8 + 2C + 8D = 0 \Rightarrow \\
(5/2)D &= (-65/8)C \Rightarrow D = \frac{-13}{4}C \\
s^3 : A + C + D &= 0 \Rightarrow A = -C - D
\end{aligned}$$

Filling these relation into the s^1: equation gives:

$$\begin{aligned}
16(-C - D) + 10(-C - 4D)/8 + C + D &= 390 \\
16(-C + (13/4)C) + 10(-C + 13C)/8 + C + (-13/4)C &= 390 \\
16(9/4)C + 15C + (-9/4)C &= 390 \\
(195/4)C = 390 \Rightarrow C = 8 \Rightarrow D = -26 \Rightarrow B = 12 \Rightarrow A = 18
\end{aligned}$$