$$U^T U = \begin{bmatrix} u_{11} & 0 & 0 \\ u_{12} & u_{22} & 0 \\ u_{13} & u_{23} & u_{33} \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} u_{11}^2 & u_{11}u_{12} & u_{11}u_{13} \\ & u_{12}^2 + u_{22}^2 & u_{12}u_{13} + u_{22}u_{23} \\ & & u_{13}^2 + u_{23}^2 + u_{33}^2 \end{bmatrix} = A$$

$$u_{11}^2 = a_{11}$$

$$u_{11}u_{12} = a_{12}$$

$$u_{11}u_{13} = a_{13}$$

$$u_{12}^2 + u_{22}^2 = a_{22}$$

$$u_{12}u_{13} + u_{22}u_{23} = a_{23}$$

$$u_{13}^2 + u_{23}^2 + u_{33}^2 = a_{33}$$

$$u_{11} = \sqrt{a_{11}}$$

$$u_{12} = \frac{a_{12}}{u_{11}}$$

$$u_{13} = \frac{a_{13}}{u_{11}}$$

$$u_{22} = \sqrt{a_{22} - u_{12}^2}$$

$$u_{23} = \frac{a_{23} - u_{12}u_{13}}{u_{22}}$$

$$u_{33} = \sqrt{a_{33} - u_{13}^2 - u_{23}^2}$$

$$A = \begin{bmatrix} 4 & 2 & 6 \\ 2 & 2 & 2 \\ 6 & 2 & 14 \end{bmatrix}$$

$$U = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix}$$

Lös $A\bar{x} = \begin{bmatrix} 0 \\ 2 \\ -6 \end{bmatrix}$

$$A\bar{x} = \begin{bmatrix} 4 & 2 & 6 \\ 2 & 2 & 2 \\ 6 & 2 & 14 \end{bmatrix}\bar{x} = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & -1 & 2 \end{bmatrix}\left(\begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix}\bar{x}\right) = \begin{bmatrix} 0 \\ 2 \\ -6 \end{bmatrix}$$

Lös $\begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & -1 & 2 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ -6 \end{bmatrix}$ $\qquad \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix}$

Lös $\begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix}$ $\qquad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

$\hat{x}$ er løsning til normalligning $\quad A^T A \hat{x} = A^T \bar{b}$

$$\hat{r} = A\hat{x} - \bar{b}$$

$$\hat{r}^T \hat{r} = (A\hat{x} - \bar{b})^T (A\hat{x} - \bar{b}) = (\hat{x}^T A^T - \bar{b}^T)(A\hat{x} - \bar{b}) =$$

$$\hat{x}^T A^T A \hat{x} - \hat{x}^T A^T \bar{b} - \bar{b}^T A \hat{x} + \bar{b}^T \bar{b} =$$

$$\hat{x}^T \underbrace{(A^T A \hat{x} - A^T \bar{b})}_{\bar{0}} - \bar{b}^T A \hat{x} + \bar{b}^T \bar{b} = -\bar{b}^T A \hat{x} + \bar{b}^T \bar{b}$$