1.

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 3 \\ 3 & 6 & 9 & 10 & 6 \\ 2 & 4 & 7 & 4 & 9 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 3 \\ 0 & 0 & 0 & -2 & -3 \\ 0 & 0 & 1 & -4 & 3 \end{array}\right] \sim$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 3 \\ 0 & 0 & 1 & -4 & 3 \\ 0 & 0 & 0 & -2 & -3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 3 \\ 0 & 0 & 1 & -4 & 3 \\ 0 & 0 & 0 & 1 & \frac{3}{2} \end{array}\right] \sim$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 0 & -3 \\ 0 & 0 & 1 & 0 & 9 \\ 0 & 0 & 0 & 1 & \frac{3}{2} \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & -30 \\ 0 & 0 & 1 & 0 & 9 \\ 0 & 0 & 0 & 1 & 3/2 \end{array}\right]$$

echelonform

$$\begin{aligned} x_1 + 2x_2 &= -30 \\ x_2 &= x_2 \\ x_3 &= 9 \\ x_4 &= \frac{3}{2} \end{aligned}$$

$$L = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -30 \\ 0 \\ 9 \\ \frac{3}{2} \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \,\middle|\, x_2 \in \mathbb{R} \right\}$$

2. $A_{3\times 2}$ $B_{3\times 2}$ $B^T_{2\times 3}$ $\bar{c}_{3\times 1}$

1. AB giver ikke mening

2. $AB^T = \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 1 \\ 4 & 0 & 2 \\ 6 & 0 & 3 \end{bmatrix}$

3. $B^TA = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 1 & 1 \end{bmatrix}$

4. $B\bar{c} =$ giver ikke mening.

3. $[A \mid I_4] =$

1.
$$\left[\begin{array}{cccc|cccc} 1 & 2 & 3 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \sim$$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 3 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{array}\right] \sim$$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 0 & 0 & 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{array}\right] \sim$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & -2 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{array}\right] = [I_4 \mid A^{-1}]$$

2. $(A^T)^{-1} = (A^{-1})^T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & -2 & 1 & 0 \end{bmatrix}$

4. 1. $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \frac{1}{2} T\left(\begin{bmatrix} 2 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

2. $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) - T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 3 \end{bmatrix} - \begin{bmatrix} 2 \\ 2 \end{bmatrix}$
$= \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

2. $A = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$

3. Nej, idet $A A^T = \begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix} \neq I_2$

5.

$$[B \mid I_3] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 2 & 2 & 1 & 0 & 0 & 1 \end{array}\right] \sim$$

1 og 2.

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 2 & 1 & -2 & 0 & 1 \end{array}\right] \sim$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] = [I_3 \mid B^{-1}]$$

$\mathcal{B}$ er basis for $\mathbb{R}^3$, idet de 3 søjler i B er lineært uafhængige (Pivot i hver søjle)

3. $$[T]_{\mathcal{B}} = B^{-1} A B$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 0 \\ 2 & 1 & 0 \\ 4 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 0 \\ -4 & -1 & 0 \\ 6 & 3 & 1 \end{bmatrix}$$

6. 1. A er trekantsmatrix $\Longrightarrow$
egenværdier = diagonalkoefficienter $\lambda_1 = 2, \lambda_2 = 1$

2. $E_2(A) = \text{Null}(A - 2I)$

$$A - 2I = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \qquad E_2(A) = \text{span}\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$$

$E_1(A) = \text{Null}(A - I)$

$$A - I = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \qquad E_1(A) = \text{span}\left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$$

3. $D = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$, $P = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

7. $\bar{v}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ $\quad \|\bar{v}_1\| = 1$

$$\bar{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \end{bmatrix} - \frac{\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \end{bmatrix}}{1} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \|\bar{v}_2\| = 1$$

$$\bar{v}_3 = \begin{bmatrix} 5 \\ 1 \\ 7 \\ 0 \end{bmatrix} - \frac{\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 1 \\ 7 \\ 0 \end{bmatrix}}{1} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} - \frac{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 1 \\ 7 \\ 0 \end{bmatrix}}{1} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} =$$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

8. 1. $C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$ $\quad C^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$$C^T C = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$(C^T C)^{-1} = \frac{1}{\det C^T C} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

$$P_W = C(C^T C)^{-1} C^T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2. $w = P_W(\overline{u}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$

$z = \overline{u} - \overline{w} = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}$

9.
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