

Opp. 1

$$1) \begin{bmatrix} 1 & -3 & 1 & 2 \\ 2 & -6 & 0 & 2 \end{bmatrix} \xrightarrow{r_2 - 2r_1 \rightarrow r_2} \begin{bmatrix} 1 & -3 & 1 & 2 \\ 0 & 0 & -2 & -2 \end{bmatrix}$$

$$\xrightarrow{r_2 / (-2)} \begin{bmatrix} 1 & -3 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{r_1 - r_2 \rightarrow r_1} \begin{bmatrix} 1 & -3 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$2) \left[ \begin{array}{cccc|c} 1 & -3 & 1 & 2 & 2 \\ 2 & -6 & 0 & 2 & 2 \end{array} \right] \rightarrow \left[ \begin{array}{cccc|c} 1 & -3 & 1 & 2 & 2 \\ 0 & 0 & -2 & -2 & -2 \end{array} \right]$$

$$\rightarrow \left[ \begin{array}{cccc|c} 1 & -3 & 1 & 2 & 2 \\ 0 & 0 & 1 & 1 & 1 \end{array} \right]$$

$$\rightarrow \left[ \begin{array}{cccc|c} \textcircled{1} & -3 & 0 & 1 & 1 \\ 0 & 0 & \textcircled{1} & 1 & 1 \end{array} \right]$$

$$x_1 = 1 + 3x_2 - x_4$$

$$x_2 = x_2$$

$$x_3 = 1 - x_4$$

$$x_4 = x_4$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

## Oppgave 2

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$$

1)  $B\vec{b}$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 0 \\ 2 \end{bmatrix}}}$$

2)  $C\vec{b}$  ej multog

$\begin{matrix} \uparrow & \uparrow \\ 3 \times 2 & 3 \times 1 \end{matrix}$

$$3) BC = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}}}$$

$$4) CB = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 3 & 2 \end{bmatrix}}}$$

Opgave 3

$$1) \left[ \begin{array}{cccc|cccc} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right] \rightarrow \left[ \begin{array}{cccc|cccc} 1 & 2 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{array} \right]$$

$$\rightarrow \left[ \begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{array} \right]$$

$$2) \det A = \det \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \underline{\underline{-1}}$$

$$\det(A^9) = (-1)^9 = \underline{\underline{-1}}$$

### Opgave 4

$$T([1]) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad T([i]) = \begin{bmatrix} 1 \\ i \end{bmatrix}$$

$$\begin{aligned} 1) \quad T([0]) &= T([1]) - T([i]) \\ &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ i \end{bmatrix} = \underline{\underline{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}} \end{aligned}$$

$$2) \quad A = [T(\bar{e}_1) \ T(\bar{e}_2)] = \underline{\underline{\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}}}$$

$$3) \quad \text{rank } A = \underline{\underline{1}} \quad \text{nullity } A = \underline{\underline{1}}$$

### opg. 5

$$1) \quad \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

den pivot i alle søjter er de lineært uafh. Der er 3 (altså nok) søjler.

$$\begin{aligned} 2) \quad \bar{u} &= 1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - 1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 2 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \\ &= \underline{\underline{\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}}} \end{aligned}$$

opgave 5 - fortsat

$$3) T(\bar{b}_1) = -\bar{b}_1$$

$$T(\bar{b}_2) = \bar{b}_2$$

$$T(\bar{b}_3) = -\bar{b}_3$$

$$[T]_B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

~~$$[T]_B u = [T]_B u = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -2 \end{bmatrix}$$~~

opgave 6

$$1) \begin{bmatrix} 2 & -3 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad \underline{\underline{\lambda_1 = 1}}$$

$$\begin{bmatrix} 2 & -3 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \quad \underline{\underline{\lambda_2 = -1}}$$

$$2) P = \underline{\underline{\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}}} \quad D = \underline{\underline{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}}$$

$$3) P^{-1} = \frac{1}{3-1} \begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 3/2 \end{bmatrix}}}$$

opgave 6 - fortsat

$$\begin{aligned} 4) \quad A^2 &= P D^2 P^{-1} = P I P^{-1} \\ &= P P^{-1} = \underline{\underline{I}} \\ A^{2012} &= P D^{2012} P^{-1} = P I P^{-1} = \underline{\underline{I}} \end{aligned}$$

opgave 7

$$S = \left\{ \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}}_{\bar{v}_1}, \underbrace{\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}}_{\bar{v}_2}, \underbrace{\begin{bmatrix} 3 \\ 1 \\ 2 \\ 4 \end{bmatrix}}_{\bar{v}_3} \right\}$$

$$1) \quad \bar{u}_1 = \underline{\underline{\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}}}$$

$$\bar{u}_2 = \bar{v}_2 - \frac{\bar{v}_2 \cdot \bar{u}_1}{\bar{u}_1 \cdot \bar{u}_1} \bar{u}_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} - \frac{1}{1} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \underline{\underline{\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}}}$$

$$\bar{u}_3 = \bar{v}_3 - \frac{\bar{v}_3 \cdot \bar{u}_1}{\bar{u}_1 \cdot \bar{u}_1} \bar{u}_1 - \frac{\bar{v}_3 \cdot \bar{u}_2}{\bar{u}_2 \cdot \bar{u}_2} \bar{u}_2$$

$$= \begin{bmatrix} 3 \\ 1 \\ 2 \\ 4 \end{bmatrix} - \frac{2}{1} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} - \frac{1}{1} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \underline{\underline{\begin{bmatrix} 3 \\ 0 \\ 0 \\ 4 \end{bmatrix}}}$$

opgave 7 - fortsat

$$2) \{ \bar{w}_1, \bar{w}_2, \bar{w}_3 \} = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3/5 \\ 0 \\ 0 \\ 4/5 \end{bmatrix} \right\}$$

opgave 8

$$W = \text{Span} \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\} \quad \bar{u} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

$$1) C^T C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$(C^T C)^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix}$$

$$C(C^T C)^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1/2 \\ 0 & 1/2 \end{bmatrix}$$

$$P_W = C(C^T C)^{-1} C^T$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1/2 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix}}}$$

$$2) \bar{w} = P_W \bar{u} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}$$

$$\bar{z} = \bar{u} - \bar{w} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} - \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}}}$$

opgave 9

$$A = \begin{bmatrix} 1 & a & b \\ 0 & 2 & c \\ 0 & 0 & 0 \end{bmatrix}$$

- rank  $A = 2$
- nullity  $A = 2$
- $A$  invertibel
- vektor uafhængig?

opgave 10       $\det A = 0$      $\det B = -1$

$$\det(B^T B) = (-1)(-1) = 1$$

$$\det((B^T B)^{-1}) = \frac{1}{1} = \underline{\underline{1}}$$

$$\det(2B) = 2^3 \det B = \underline{\underline{-8}}$$

$$\det(B^T A B) = \underline{\underline{0}}$$

opgave 11

✓  $\vec{v}_3 \in W$

✓  $\det A = 0$

✓ 0 er egenverdi for  $A$

opgave 12

T	F
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# Point oversigt

Opgave	spørgsmål	Point
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1	1	3
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	2	5
--	---	---

2	1	1
---	---	---

	2	$1\frac{1}{2}$
--	---	----------------

	3	$1\frac{1}{2}$
--	---	----------------

	4	2
--	---	---

3	1	5
---	---	---

	2	$2+2=4$
--	---	---------

4	1	3
---	---	---

	2	3
--	---	---

	3	$1\frac{1}{2}+1\frac{1}{2}=3$
--	---	-------------------------------

5	1	4
---	---	---

	2	3
--	---	---

	3	3
--	---	---

6	1	$2\frac{1}{2}$
---	---	----------------

	2	$2\frac{1}{2}$
--	---	----------------

	3	2
--	---	---

	4	$1\frac{1}{2}+1\frac{1}{2}=3$
--	---	-------------------------------

7	1	8
---	---	---

	2	2
--	---	---

8	1	6
---	---	---

	2	$2+2=4$
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