

Cylinder-koordinater

$$x = r \cos \theta, \quad y = r \sin \theta, \quad z = z, \quad dV = r \, dz \, dr \, d\theta$$

Simpelt område:

$$T = \{(r, \theta, z) : \theta_1 \leq \theta \leq \theta_2, r_1(\theta) \leq r \leq r_2(\theta), z_1(r, \theta) \leq z \leq z_2(r, \theta)\}$$

Så gælder

$$\begin{aligned} \iiint_T f(x, y, z) \, dV \\ = \int_{\theta_1}^{\theta_2} \int_{r_1(\theta)}^{r_2(\theta)} \int_{z_1(r, \theta)}^{z_2(r, \theta)} f(r \cos \theta, r \sin \theta, z) r \, dz \, dr \, d\theta \end{aligned}$$

Sfæriske-koordinater

$$x = \rho \sin \phi \cos \theta$$

$$y = \rho \sin \phi \sin \theta$$

$$z = \rho \cos \phi$$

$$dV = \rho^2 \sin \phi d\rho d\phi d\theta$$

Sfæriske-koordinater

Simpelt område:

$$T = \{(\rho, \phi, \theta) : \theta_1 \leq \theta \leq \theta_2, \phi_1 \leq \phi \leq \phi_2, \\ \rho_1(\phi, \theta) \leq \rho \leq \rho_2(\phi, \theta)\}.$$

Så gælder

$$\iiint_T f(x, y, z) dV = \\ \int_{\theta_1}^{\theta_2} \int_{\phi_1}^{\phi_2} \int_{\rho_1(\phi, \theta)}^{\rho_2(\phi, \theta)} f(\rho \sin \phi \cos \theta, \rho \sin \phi \sin \theta, \rho \cos \phi) \rho^2 \sin \phi d\rho d\phi d\theta$$