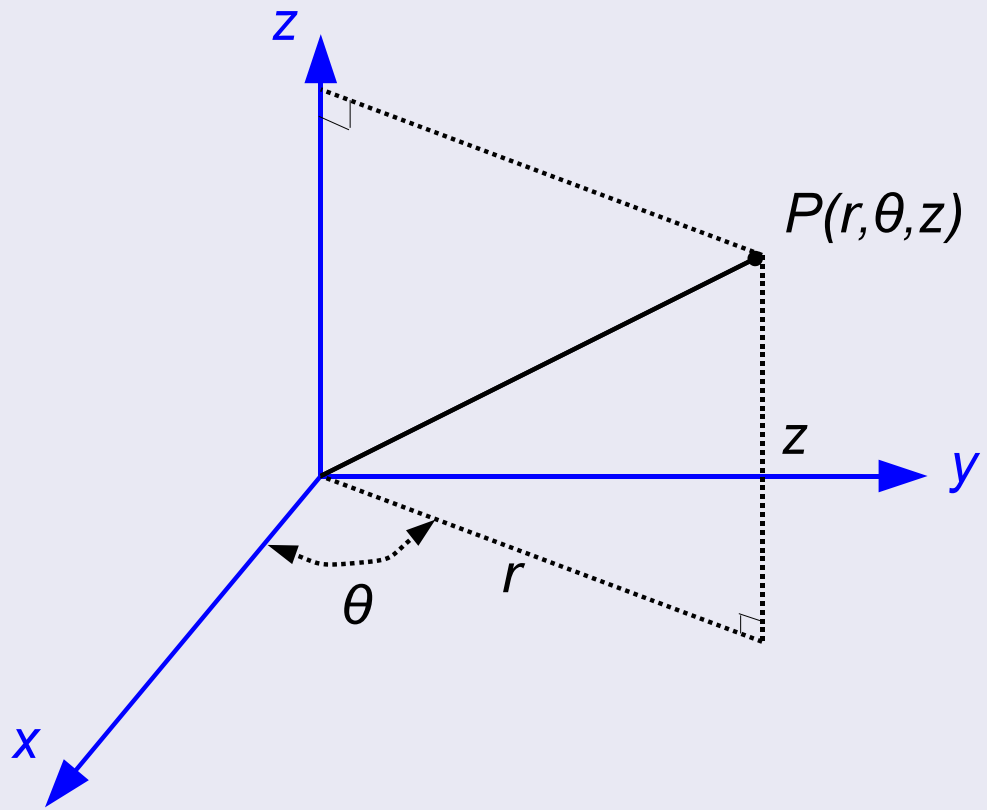
Cylinder-koordinater

Koordinater og volumenelement



$$x = r\cos\theta, \quad y = r\sin\theta, \quad z = z, \quad dV = r\,dz\,dr\,d\theta$$

Simpelt område mht. cylinder-koordinater

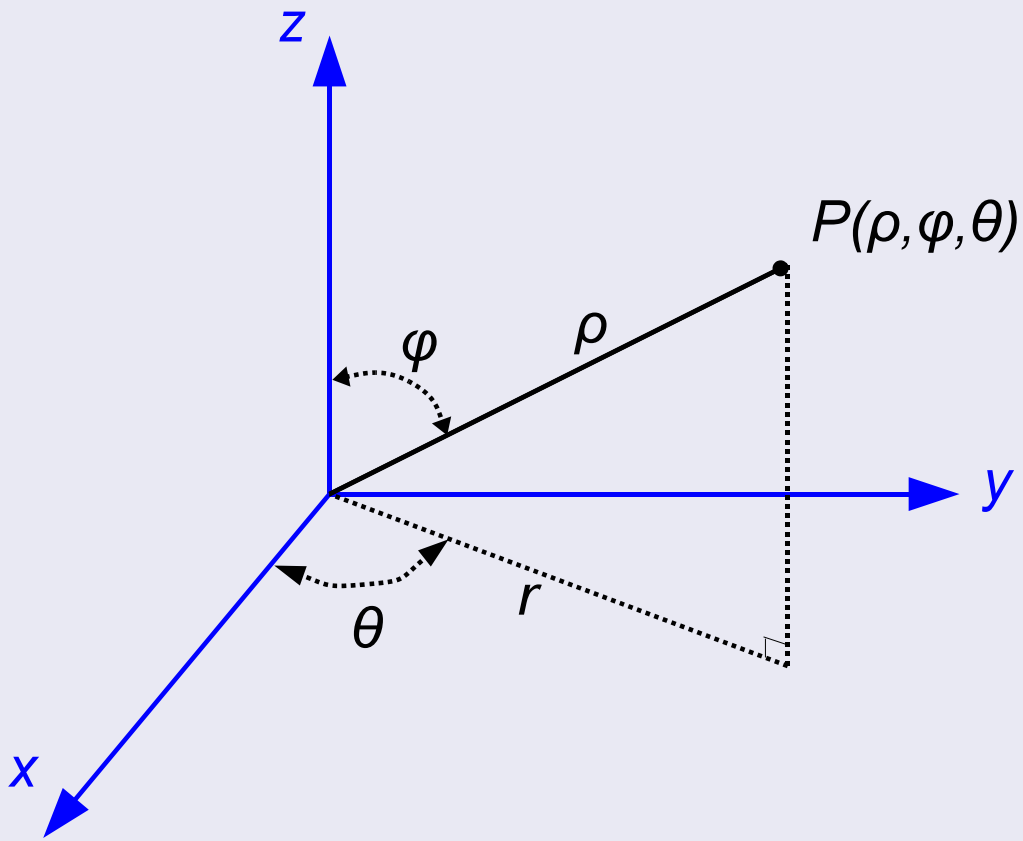
$$T = \{(r,\theta,z) : \theta_1 \le \theta \le \theta_2, r_1(\theta) \le r \le r_2(\theta), z_1(r,\theta) \le z \le z_2(r,\theta)\}.$$

Så gælder

$$\iiint_T f(x,y,z)\,dV = \int_{\theta_1}^{\theta_2} \int_{r_1(\theta)}^{r_2(\theta)} \int_{z_1(r,\theta)}^{z_2(r,\theta)} f(r\cos\theta, r\sin\theta, z) r\,dz\,dr\,d\theta$$

Sfæriske-koordinater

Koordinater og volumenelement



$$x = \rho \sin\varphi \cos\theta \qquad y = \rho \sin\varphi \sin\theta$$

$$z = \rho \cos\varphi \qquad dV = \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

Sfærisk simpelt område:

$$T = \{(\rho, \varphi, \theta) : \theta_1 \leq \theta \leq \theta_2, \varphi_1 \leq \varphi \leq \varphi_2, \\ \rho_1(\varphi, \theta) \leq \rho \leq \rho_2(\varphi, \theta)\}.$$

Så gælder

$$\iiint_T f(x, y, z)\, dV = \\ \int_{\theta_1}^{\theta_2} \int_{\varphi_1}^{\varphi_2} \int_{\rho_1(\varphi,\theta)}^{\rho_2(\varphi,\theta)} f(\rho \sin\varphi \cos\theta, \rho \sin\varphi \sin\theta, \rho \cos\varphi)\rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta$$