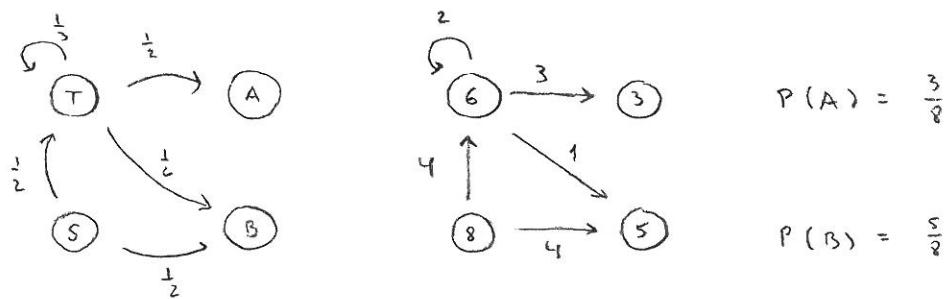


# SFK

1.7

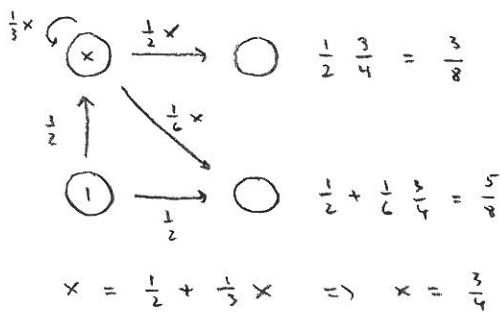


$$P(A) = \frac{3}{8}$$

$$P(B) = \frac{5}{8}$$

$$\text{gns. tid : } \frac{8+6}{3+5} = \frac{7}{4} = 1,75$$

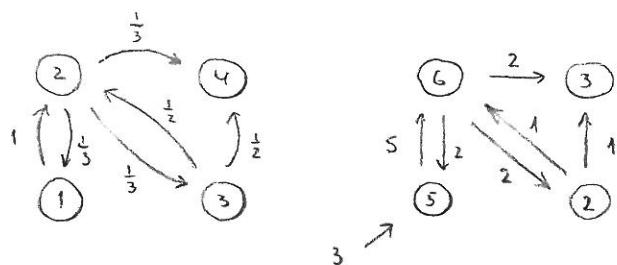
gennemstrømmingsmetoden :



$$\text{gns. tid : } 1 + \frac{3}{4} = \frac{7}{4} = 1,75$$

$$x = \frac{1}{2} + \frac{1}{3}x \Rightarrow x = \frac{3}{4}$$

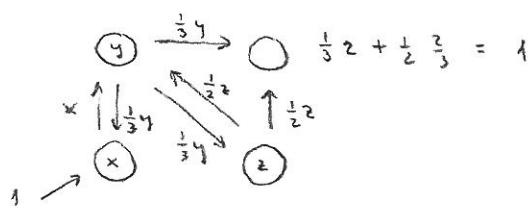
1.8



gns. tid :

$$\frac{5+6+2}{3} = \frac{13}{3} \approx 4,33$$

gennemstrømmingsmetoden :



$$x = 1 + \frac{1}{3}y$$

$$\left. \begin{array}{l} y = x + \frac{1}{2}z \\ z = \frac{1}{3}y \end{array} \right\} \quad \left. \begin{array}{l} y = x + \frac{1}{2}y, \quad x = \frac{5}{6}y \\ \frac{5}{6}y = 1 + \frac{1}{3}y, \quad y = 2 \end{array} \right\}$$

$$x = \frac{5}{3}$$

$$z = \frac{2}{3}$$

$$\text{gns. tid : } x+y+z = \frac{5}{3} + 2 + \frac{2}{3} = \frac{13}{3} \approx 4,33$$

matrixmetoden :

$$Q = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{2} & 0 \end{bmatrix}, \quad E - Q = \begin{bmatrix} 1 & -1 & 0 \\ -\frac{1}{3} & 1 & -\frac{1}{3} \\ 0 & -\frac{1}{2} & 1 \end{bmatrix}, \quad \det(E - Q) = \frac{5}{6} + (-\frac{1}{3}) = \frac{1}{2}$$

fortsetter

1.8 portrat

$$(E - Q)^{-1} = \frac{1}{2} \begin{bmatrix} \frac{5}{6} & \frac{1}{3} & \frac{1}{6} \\ 1 & 1 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & \frac{2}{3} \end{bmatrix}^T = 2 \begin{bmatrix} \frac{5}{6} & 1 & \frac{1}{3} \\ \frac{1}{3} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{2}{3} \end{bmatrix}, N_1 = 2 \left( \frac{5}{6} + 1 + \frac{1}{3} \right) = \frac{13}{3} \approx 4,33$$

4.3

$$P = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}, E = [1], R = \begin{bmatrix} \frac{1}{3} \\ 0 \end{bmatrix}, Q = \begin{bmatrix} \frac{2}{3} & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\text{ii)} \quad Q^2 = \begin{bmatrix} \frac{2}{3} & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{2}{3} & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{4}{9} & 0 \\ \frac{3}{4} & \frac{1}{4} \end{bmatrix}$$

$$Q^3 = \begin{bmatrix} \frac{2}{3} & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{4}{9} & 0 \\ \frac{3}{4} & \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{8}{27} & 0 \\ \frac{37}{72} & \frac{1}{8} \end{bmatrix}$$

$$Q^4 = \begin{bmatrix} \frac{2}{3} & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{8}{27} & 0 \\ \frac{37}{72} & \frac{1}{8} \end{bmatrix} = \begin{bmatrix} \frac{16}{81} & 0 \\ \frac{175}{432} & \frac{1}{16} \end{bmatrix}$$

$$(E + Q)R = \begin{bmatrix} \frac{5}{3} & 0 \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{5}{9} \\ \frac{1}{6} \end{bmatrix}$$

$$(E + Q + Q^2)R = \begin{bmatrix} \frac{19}{9} & 0 \\ \frac{13}{12} & \frac{7}{4} \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{19}{27} \\ \frac{13}{36} \end{bmatrix}$$

$$(E + Q + Q^2 + Q^3)R = \begin{bmatrix} \frac{65}{27} & 0 \\ \frac{115}{72} & \frac{15}{8} \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{65}{81} \\ \frac{115}{216} \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 1 & 0 & 0 \\ \frac{5}{9} & \frac{4}{9} & 0 \\ \frac{1}{6} & \frac{7}{12} & \frac{1}{4} \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 1 & 0 & 0 \\ \frac{19}{27} & \frac{8}{27} & 0 \\ \frac{13}{36} & \frac{37}{72} & \frac{1}{8} \end{bmatrix}$$

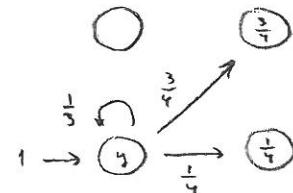
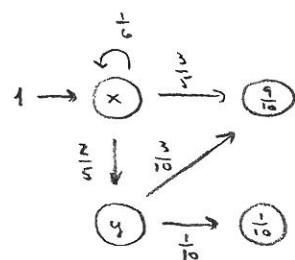
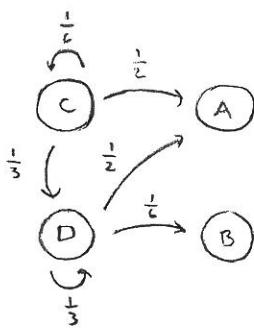
$$P^4 = \begin{bmatrix} 1 & 0 & 0 \\ \frac{65}{81} & \frac{16}{81} & 0 \\ \frac{115}{216} & \frac{175}{432} & \frac{1}{16} \end{bmatrix}$$

$$\text{iii)} \quad \lim_{n \rightarrow \infty} P^n = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \text{ idet } \lim_{n \rightarrow \infty} Q^n = 0$$

$$\text{Kontrollrechnungen: } E - Q = \begin{bmatrix} \frac{1}{3} & 0 \\ -\frac{1}{3} & \frac{1}{3} \end{bmatrix}; \det(E - Q) = \frac{1}{6}$$

$$(E - Q)^{-1} = \frac{1}{6} \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \end{bmatrix}^T = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}, B = (E - Q)^{-1} e = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

4.4



$$x = 1 + \frac{1}{6}x \Rightarrow x = \frac{6}{5}$$

$$y = \frac{2}{5} + \frac{1}{3}y \Rightarrow y = \frac{3}{5}$$

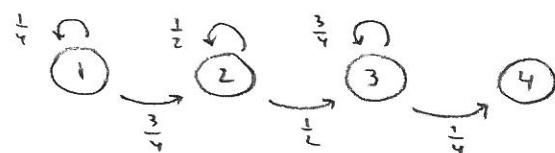
$$y = 1 + \frac{1}{3}y \Rightarrow y = \frac{3}{2}$$

$$N = \begin{bmatrix} \frac{6}{5} & \frac{3}{5} \\ 0 & \frac{3}{2} \end{bmatrix}, \quad N_C = \frac{6}{5} + \frac{3}{5} = \frac{9}{5}$$

$$N_D = 0 + \frac{3}{2} = \frac{3}{2}$$

$$\text{kontrolregning: } B = NR = \begin{bmatrix} \frac{6}{5} & \frac{3}{5} \\ 0 & \frac{3}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{9}{10} & \frac{1}{10} \\ \frac{3}{4} & \frac{1}{4} \end{bmatrix}$$

4.5



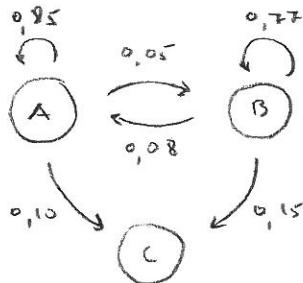
# forskellige farver

$$1 \rightarrow \left(\frac{4}{3}\right) \rightarrow 2 \rightarrow 4 \rightarrow 1$$

$$N = \frac{4}{3} + 2 + 4 = \frac{22}{3}$$

$$\approx 7,33$$

4.6



$$Q = \begin{bmatrix} 0,85 & 0,05 \\ 0,08 & 0,77 \end{bmatrix}$$

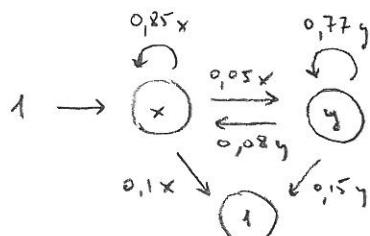
$$E - Q = \begin{bmatrix} 0,15 & -0,05 \\ -0,08 & 0,23 \end{bmatrix}$$

$$\det(E - Q) = 0,0305 = \frac{61}{2000}$$

$$N = \frac{2000}{61} \begin{bmatrix} 0,23 & 0,08 \\ 0,05 & 0,15 \end{bmatrix}^T = \frac{1}{61} \begin{bmatrix} 460 & 100 \\ 160 & 300 \end{bmatrix}$$

$$N_A = \frac{1}{61} (460 + 100) = \frac{560}{61} \approx 9,18 \text{ under.}$$

gennemstrømmingsmetoden:



$$x = 1 + \frac{85}{100}x + \frac{8}{100}y$$

$$y = \frac{5}{100}x + \frac{77}{100}y, \quad y = \frac{5}{23}x$$

$$\frac{3}{20}x = 1 + \frac{4}{230}x, \quad x = \frac{460}{61}, \quad y = \frac{100}{61}$$

$$N_A = x + y = \frac{560}{61} = 9,18 \text{ under.}$$

SFK

4.7

$$1) \quad N = (E - Q)^{-1} \Rightarrow N^{-1} = E - Q \Rightarrow Q = E - N^{-1}$$

$$2) \quad NQ = NE - NN^{-1} = N - E$$

4.8

Trefansandsyndigheder  $A: \frac{2}{3}$   $B: \frac{1}{2}$   $C: \frac{1}{3}$

1)  $AB$  ikke mulig, da ingen skyder på  $C$  før  $A$  eller  $B$  er elimineret, alle andre kombinationer er mulige, dvs.

$$T = \{ABC, AC, BC, A, B, C, \emptyset\}$$

$$2) \quad P(ABC \rightarrow A|BC) = \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{9}$$

$$P(ABC \rightarrow AC) = \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{2}{3} = \frac{2}{9}$$

$$P(ABC \rightarrow BC) = \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{2}{3} + \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{3} = \frac{2}{9}$$

$$P(ABC \rightarrow C) = \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{2}{3} + \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{3} + \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{3} + \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{3} = \frac{4}{9}$$

$$P(ABC \rightarrow \emptyset) = 0$$

$$P(AC \rightarrow AC) = \frac{1}{3} \cdot \frac{2}{3} = \frac{2}{9}$$

$$P(\emptyset \rightarrow BC) = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$$

$$P(AC \rightarrow A) = \frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9}$$

$$P(BC \rightarrow B) = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$$

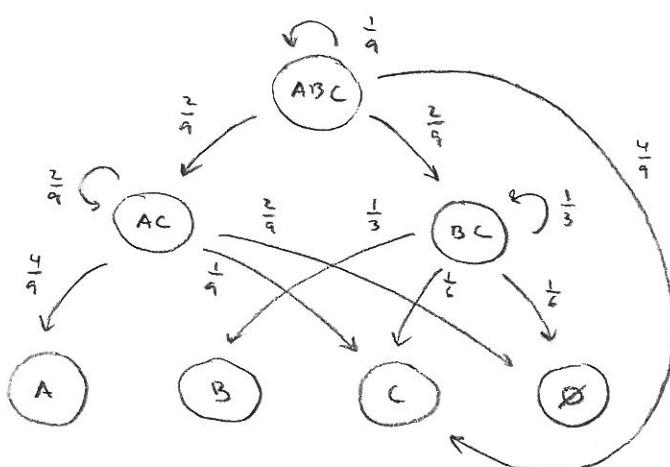
$$P(AC \rightarrow C) = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$$

$$P(BC \rightarrow C) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

$$P(AC \rightarrow \emptyset) = \frac{2}{3} \cdot \frac{1}{3} = \frac{2}{9}$$

$$P(BC \rightarrow \emptyset) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

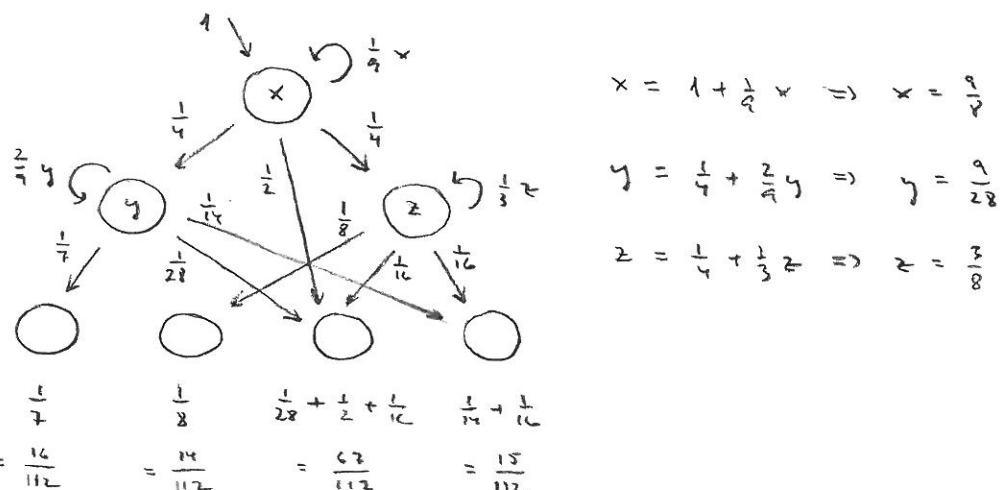
$$P = \begin{bmatrix} 0 & A & B & C & AC & BC & ABC \\ \begin{matrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{2}{9} & \frac{4}{9} & 0 & \frac{1}{9} & \frac{3}{9} & 0 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{4}{9} & \frac{2}{9} & \frac{2}{9} & \frac{1}{9} \end{matrix} \end{bmatrix} \begin{matrix} \emptyset \\ A \\ B \\ C \\ AC \\ BC \\ ABC \end{matrix}$$



fortsatte

4.8 fortset

3)



4)

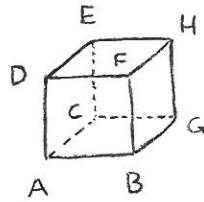
Forventet varighed af tankeslægt:

$$x + y + z = \frac{9}{2} + \frac{9}{28} + \frac{3}{8} = \frac{51}{28} \approx 1,82$$

Forventet antal afgivende skud:

$$3 \cdot \frac{9}{8} + 2 \left( \frac{9}{28} + \frac{3}{8} \right) = \frac{267}{56} \approx 4,77$$

7.2



Random walk på hjørnerne af terning

Antag, at A og B er absorberende

tilstande

i Sandsynighed for absorption i A

- ved start i C eller D:

$$r_1 = \frac{1}{3} r_1 + \frac{1}{3} r_2 + \frac{1}{3} r_3 \Leftrightarrow 3r_1 - r_2 - r_3 = 1$$

- ved start i E:

$$r_2 = \frac{2}{3} r_1 + \frac{1}{3} r_4 \Leftrightarrow 2r_1 - 3r_2 + r_4 = 0$$

- ved start i F eller G:

$$r_3 = \frac{1}{3} 0 + \frac{1}{3} r_1 + \frac{1}{3} r_4 \Leftrightarrow r_1 - 3r_3 + r_4 = 0$$

- ved start i H:

$$r_4 = \frac{1}{3} r_2 + \frac{2}{3} r_3 \Leftrightarrow r_2 + 2r_3 - 3r_4 = 0$$

$$\left[ \begin{array}{cccc|c} 3 & -1 & -1 & 0 & 1 \\ 2 & -3 & 0 & 1 & 0 \\ 1 & 0 & -3 & 1 & 0 \\ 0 & 1 & 2 & -3 & 0 \end{array} \right] \sim \dots \sim \left[ \begin{array}{cccc|c} 1 & 0 & -3 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 2 & -4 & -1 \\ 0 & 0 & 0 & 7 & 3 \end{array} \right]$$

$$(r_1, r_2, r_3, r_4) = \left( \frac{9}{14}, \frac{4}{7}, \frac{5}{14}, \frac{3}{7} \right)$$

ii Forventet tid til absorption i A eller B

- ved start i C, D, F eller G:

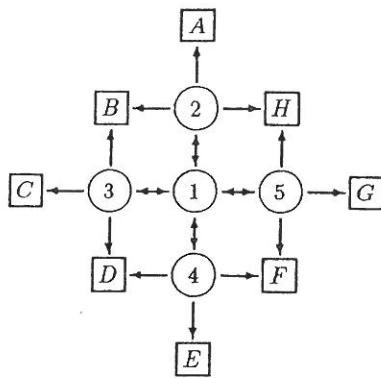
$$\mu_1 = 1 + \frac{2}{3} \mu_1 + \frac{1}{3} \mu_2 \Leftrightarrow \mu_1 = \frac{3}{2} + \frac{1}{2} \mu_2$$

- ved start i E eller H:

$$\mu_2 = 1 + \frac{2}{3} \mu_1 + \frac{1}{3} \mu_2 \Leftrightarrow \mu_2 = \frac{3}{2} + \mu_1$$

$$\mu_1 = \frac{3}{2} + \frac{1}{2} \left( \frac{3}{2} + \mu_1 \right) \Rightarrow \mu_1 = \frac{9}{2} \Rightarrow \mu_2 = 6$$

$$(\mu_1, \mu_2) = \left( \frac{9}{2}, 6 \right)$$



Random walk på gitter

□ markører absorberende tilstande

○ markører transiente tilstande

i Absorptionsandsynigheder :

$$r_{1A} = \frac{1}{4} r_{2A} + \frac{3}{4} r_{3A} \quad r_{1B} = \frac{1}{2} r_{2B} + \frac{1}{2} r_{4B}$$

$$r_{2A} = \frac{1}{4} + \frac{1}{4} r_{2A} \quad r_{2B} = \frac{1}{4} + \frac{1}{4} r_{1B}$$

$$r_{3A} = \frac{1}{4} r_{1A} \quad r_{4B} = \frac{1}{4} r_{1B}$$

$$r_{1A} = \frac{1}{4} \left( \frac{1}{4} + \frac{1}{4} r_{1A} \right) + \frac{3}{4} \frac{1}{4} r_{1A} \Rightarrow r_{1A} = \frac{1}{12}$$

$$r_{1B} = \frac{1}{2} \left( \frac{1}{4} + \frac{1}{4} r_{1B} \right) + \frac{1}{2} \frac{1}{4} r_{1B} \Rightarrow r_{1B} = \frac{1}{6}$$

$$(r_{1A}, r_{2A}, r_{3A}) = \left( \frac{1}{12}, \frac{13}{48}, \frac{1}{48} \right)$$

$$(r_{1B}, r_{2B}, r_{4B}) = \left( \frac{1}{6}, \frac{7}{24}, \frac{1}{24} \right)$$

ii Forventet tid til absorption

- ved start i tilstand 1 :  $\mu_1 = 1 + \mu_2$ - ved start i tilstand 2 :  $\mu_2 = 1 + \frac{1}{4} \mu_1$ 

$$\mu_1 = 1 + 1 + \frac{1}{4} \mu_1 \Rightarrow \mu_1 = \frac{8}{3} \Rightarrow \mu_2 = \frac{5}{3}$$

$$(\mu_1, \mu_2) = \left( \frac{8}{3}, \frac{5}{3} \right)$$

Øvrige absorptionsandsynigheder og forventede tider får ved symmetri-betrægninger.