

Vink til opgavesæt 20

8.4

40 a Mellemresultat:

$$Y_{i,i+1} = i+2, \quad Y_{i,i-1} = 3i, \quad i = 0, 1, \dots$$

b

$$\text{Bemerk, at } \frac{\pi_{10,11}}{\pi_{10,9} + \pi_{10,11}} = \frac{Y_{10,11}}{Y_{10,9} + Y_{10,11}}$$

c Husk, at $E_0[\tau_0] = \frac{1}{\pi_0}$.

41 Rækkeoperationer

$$G^T = \begin{bmatrix} -1 & 3 & 0 & 0 & \dots \\ 1 & -4 & 6 & 0 & \dots \\ 0 & 1 & -8 & 9 & \dots \\ 0 & 0 & 2 & -12 & \dots \\ \vdots & \vdots & \vdots & \vdots & \end{bmatrix} = \begin{bmatrix} -1 & 3 & 0 & 0 & \dots \\ 0 & -1 & 6 & 0 & \dots \\ 0 & 0 & -2 & 9 & \dots \\ 0 & 0 & 0 & -3 & \dots \\ \vdots & \vdots & \vdots & \vdots & \end{bmatrix}$$

43 a, b Benyt eks. 8.33 side 493.

44 a M/M/1/1, $\rho = 1$

b M/M/1/1, $\rho = \frac{1}{2}$

c M/M/1/2, $\rho = 1$

d M/M/2/2, $\rho = \frac{1}{2}$

45 a Rækkeoperationer

$$G^T = \begin{bmatrix} -1 & 2 & 0 & 0 \\ 0 & -3 & 2 & 0 \\ 1 & 0 & -2 & 2 \\ 0 & 1 & 0 & -2 \end{bmatrix} \sim \dots \sim \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

b Rækkeoperationer

$$G^T = \begin{bmatrix} -1 & 2 & 0 & 0 \\ 0 & -3 & 2 & 0 \\ 1 & 0 & -\frac{5}{2} & 2 \\ 0 & 1 & \frac{1}{2} & -2 \end{bmatrix} \sim \dots \sim \begin{bmatrix} 7 & 0 & 0 & -16 \\ 0 & 7 & 0 & -8 \\ 0 & 0 & 7 & -12 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

46 a -

b Rækkeoperationer

$$G^T = \begin{bmatrix} -1 & 1 & 0 & 0 \\ \frac{1}{2} & -2 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{3}{2} & 1 \\ 0 & \frac{1}{2} & \frac{1}{2} & -1 \end{bmatrix} \sim \dots \sim \begin{bmatrix} 5 & 0 & 0 & -4 \\ 0 & 5 & 0 & -4 \\ 0 & 0 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

47 a Mellemlæsning: $\gamma_{12} = 2,1$

Rækkeoperationer

$$G^T = \begin{bmatrix} -3 & 0,5 & 0 \\ 3 & -2,6 & 0,5 \\ 0 & 2,1 & -0,5 \end{bmatrix} \sim \dots \sim \begin{bmatrix} 6 & -1 & 0 \\ 0 & 21 & -5 \\ 0 & 0 & 0 \end{bmatrix}$$

b -

48 Mellemlæsning: $\sum_{m=1}^{\infty} \frac{\lambda_0 \dots \lambda_{n-1}}{M_1 \dots M_n} = e^{\frac{\lambda}{m}} - 1$

49 -

51 Bemerk, at $T_n = \sum_{j=0}^n V_i$, $V_j \sim e(\mu)$, $j = 0, \dots, n$,
med.

$$\Rightarrow T_n \sim \Gamma(n+1, \mu)$$

Udregn $P(T > t) = \sum_{n=0}^{\infty} P(T_n > t) P(X_t = n)$ ved

$$\text{udnyttelse af } P(T_n > t) = e^{-\mu t} \sum_{k=0}^{\infty} \frac{(\mu t)^k}{k!}, \text{ jf.}$$

formel side 134 midt

$$\text{Mellemlæsning: } P(T > t) = e^{-(1-\varrho)\mu t}$$