

## Egen værdier og egenvektorer

$T: V \rightarrow V$  lineær operator på  $V$

Hvis der findes en skalar  $\lambda$  og en vektor  $\bar{x} \neq \bar{0}$ , så

$$T(\bar{x}) = \lambda \bar{x},$$

dvs. så villedet af  $\bar{x}$  er proportionalt med  $\bar{x}$  selv, så kaldes

$\lambda$  en egen værdi for  $T$ , og

$\bar{x}$  en egenvektor hørende til  $\lambda$ .

Bemerk

$\bar{x}$  egenvektor  $\Rightarrow r\bar{x}$ ,  $r \neq 0$ , egenvektor

eks.  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ ,  $T(x,y) = (y,x)$

$$(y,x) = \lambda(x,y), (x,y) \neq (0,0)$$

$$\Leftrightarrow y = \lambda x \wedge x = \lambda y, (x,y) \neq (0,0)$$

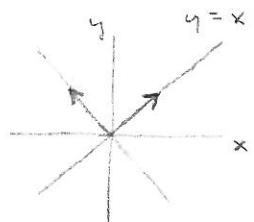
$$\Leftrightarrow \lambda^2 = 1 \Leftrightarrow \lambda = \pm 1$$

$$\lambda = 1: (y,x) = (x,y) \Leftrightarrow y = x$$

dvs.  $r(1,1)$ ,  $r \neq 0$ , er egenvektorer

$$\lambda = -1: (y,x) = (-x,-y) \Leftrightarrow y = -x$$

dvs.  $s(-1,1)$ ,  $s \neq 0$ , er egenvektorer



eks.  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ :  $T(x,y,z) = (zy, 0, sz)$

$$(zy, 0, sz) = \lambda(x,y,z), (x,y,z) \neq (0,0,0)$$

$$\Leftrightarrow zy = \lambda x \wedge 0 = \lambda y \wedge sz = \lambda z, (x,y,z) \neq (0,0,0)$$

$$(1) \quad \lambda = 0 \wedge y = 0 \wedge z = 0,$$

dvs.  $(x,y,z) = r(1,0,0)$  er egenvektorer



$$(2) \quad \lambda = \pm 1 \wedge y = 0 \wedge z = 0,$$

dvs.  $(x,y,z) = s(0,0,1)$  er egenvektorer

En mere systematisk metode

$$T: \mathbb{R}^n \rightarrow \mathbb{R}^n, \quad T(\bar{x}) = A\bar{x}$$

$$T(\bar{x}) = \lambda \bar{x} \Leftrightarrow A\bar{x} = \lambda \bar{x} \Leftrightarrow A\bar{x} - \lambda I\bar{x} = 0 \Leftrightarrow (A - \lambda I)\bar{x} = 0$$

Nødvendig betingelse for egentlig løsninger:

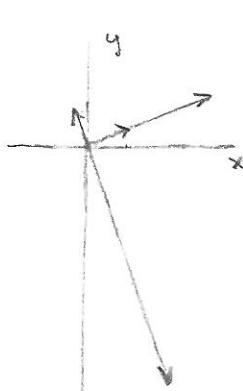
$\det(A - \lambda I) = 0$ , kaldes den karakteristiske ligning

$\det(A - \lambda I)$  kaldes det karakteristiske polynomium

Eigenverdiene findes ved at løse  $\det(A - \lambda I) = 0$ .

Eigenvektorer hørende til  $\lambda$  findes ved at løse det homogene ligningsystem  $(A\bar{x} - \lambda I)\bar{x} = 0$ , og se bort fra nulløsningen.

eks.  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2, \quad T(\bar{x}) = A\bar{x}, \quad A = \begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}$



$$\begin{aligned} \det(A - \lambda I) &= \begin{vmatrix} 2-\lambda & 3 \\ 3 & -6-\lambda \end{vmatrix} = (2-\lambda)(-6-\lambda) - 9 \\ &= -12 - 2\lambda + 6\lambda + \lambda^2 - 9 = \lambda^2 + 4\lambda - 21 = 0 \\ d &= 16 + 84 = 100, \quad \lambda = \frac{-4 \pm \sqrt{100}}{2} = \left\{ \begin{array}{l} -7 \\ 3 \end{array} \right. \end{aligned}$$

$$\lambda = -7: \quad \begin{bmatrix} 9 & 3 \\ 3 & 1 \end{bmatrix} \sim \begin{bmatrix} 3 & 1 \\ 0 & 0 \end{bmatrix}; \quad \bar{x} = r \begin{bmatrix} -1 \\ 3 \end{bmatrix}, \quad r \neq 0$$

$$\lambda = 3: \quad \begin{bmatrix} -1 & 3 \\ 3 & -9 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 \\ 0 & 0 \end{bmatrix}; \quad \bar{x} = s \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \quad s \neq 0$$

eks.  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3, \quad T(\bar{x}) = A\bar{x}, \quad A = \begin{bmatrix} 3 & 6 & -8 \\ 0 & 0 & 6 \\ 0 & 0 & 2 \end{bmatrix}$

$$\det(A - \lambda I) = \begin{vmatrix} 3-\lambda & 6 & -8 \\ 0 & -\lambda & 6 \\ 0 & 0 & 2-\lambda \end{vmatrix} = -\lambda(\lambda-2)(\lambda-3) = 0$$

for  $\lambda = \left\{ \begin{array}{l} 0 \\ 2 \\ 3 \end{array} \right.$

$$\lambda = 0: \quad \begin{bmatrix} 3 & 6 & -8 \\ 0 & 0 & 6 \\ 0 & 0 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}; \quad \bar{x} = r \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \quad r \neq 0$$

$$\lambda = 2 : \begin{bmatrix} 1 & 0 & -8 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}; \quad \bar{x} = s \begin{bmatrix} -10 \\ 3 \\ 1 \end{bmatrix}, s \neq 0$$

$$\lambda = 3 : \begin{bmatrix} 0 & 6 & -8 \\ 0 & -3 & 0 \\ 0 & 0 & -1 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad \bar{x} = t \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, t \neq 0$$

dann  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ ,  $T(\bar{x}) = A\bar{x}$ ,  $A = \begin{bmatrix} -\frac{1}{2}-x & \frac{1}{2} & \frac{2}{3} \\ \frac{1}{2} & \frac{2}{3}-x & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{2} & \frac{1}{2}-x \end{bmatrix}$

$$\begin{bmatrix} \frac{1}{2}-x & \frac{1}{2} & \frac{2}{3} \\ \frac{1}{2} & \frac{2}{3}-x & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{2} & \frac{1}{2}-x \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}-x & \frac{1}{2} & \frac{2}{3} \\ 0 & \frac{2}{3}-x & \frac{1}{2} \\ \frac{1}{2}+x & \frac{1}{2} & \frac{1}{2}-x \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2} & \frac{2}{3}-x \\ 0 & \frac{2}{3}-x & \frac{1}{2} \\ \frac{1}{2}+x & \frac{1}{2} & \frac{1}{2}-x \end{bmatrix}$$

$$= \left( \frac{1}{2}+x \right) \left( \frac{1}{18} - \left( \frac{2}{3}-x \right) \left( \frac{5}{2}-x \right) \right) = -(\lambda + \frac{1}{2})(\lambda^2 - \frac{3}{2}\lambda + \frac{1}{2})$$

$$= -(\lambda + \frac{1}{2})(\lambda - \frac{1}{2})(\lambda - 1) = 0 \quad \text{für } \lambda = -\frac{1}{2}, \frac{1}{2}, 1$$

$$\lambda = -\frac{1}{2}:$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

$$\bar{x} = r(-1, 0, 1), r \neq 0$$

$$\lambda = \frac{1}{2}:$$

$$\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{2}{3} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{2} & \frac{1}{2}-x \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 \\ -2 & 1 & 4 \\ 4 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & 6 \\ 0 & -3 & -6 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix};$$

$$\bar{x} = s(1, -2, 1), s \neq 0$$

$$\lambda = 1:$$

$$\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{2}{3} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 1 \\ -5 & 1 & 4 \\ 4 & 1 & -5 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 1 \\ 0 & 9 & 9 \\ 0 & 9 & -9 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix};$$

$$\bar{x} = t(1, 1, 1), t \neq 0$$

Bemerkung 0 er egenwert von  $\Rightarrow T$  ist nicht bijektiv

Nur  $T: \mathbb{R}^n \rightarrow \mathbb{R}^n \Rightarrow A$  er singular

Nogle definitioner:

Eigenrummet hørende til  $\lambda$ :  $\text{Ker}(T - \lambda I)$

Når  $T: \mathbb{R}^n \rightarrow \mathbb{R}^n$ :  $\text{Null}(A - \lambda I)$

~ alle egenvektorer forenet med  $\vec{0}$

Den algebraiske multiplicitet af  $\lambda$ : # gange

$\lambda$  er rod i det karakteristiske polynomium.

Den geometriske multiplicitet af  $\lambda$ : Dimensionen af eigenrummet hørende til  $\lambda$ .

Sætn. u/ bevis:

For en given egenveordi gælder

1 ≤ geometrisk mult. ≤ algebraisk mult.

eks.  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ ,  $T(\vec{x}) = A\vec{x}$ ,  $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix}$

$$\begin{vmatrix} -\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ 3 & 0 & 1-\lambda \end{vmatrix} = -\lambda^2(\lambda-1) = 0 \text{ for } \lambda = \begin{cases} 0 & (\text{dobbeltrod}) \\ 1 \end{cases}$$

$$\lambda=0: \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 3 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad \vec{x} = r \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix} + s \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad (r,s) \neq (0,0)$$

$$\lambda=1: \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 3 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad \vec{x} = t \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad t \neq 0$$

$\lambda=0$  har alg. mult. 2 og geom. mult. 2

eks.  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ ,  $T(\vec{x}) = A\vec{x}$ ,  $A = \begin{bmatrix} 4 & 0 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

$$\begin{vmatrix} 4-\lambda & 0 & 0 \\ 1 & 4-\lambda & 0 \\ 0 & 0 & 5-\lambda \end{vmatrix} = -(4-\lambda)^2(5-\lambda) = 0 \quad \lambda = \begin{cases} 4 & (\text{dobbeltrod}) \\ 5 \end{cases}$$

$$\lambda = 4 : \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}; \quad \bar{x} = r \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, r \neq 0$$

$$\lambda = 5 : \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad \bar{x} = s \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, s \neq 0$$

$\lambda = 4$  har alg. mult. 2 og geom. mult. 1

eks.  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ ,  $T(\bar{x}) = A\bar{x}$ ,  $A = \begin{bmatrix} 5 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & 1 & 5 \end{bmatrix}$

$$\begin{vmatrix} 5-\lambda & 0 & 0 \\ 1 & 5-\lambda & 0 \\ 0 & 1 & 5-\lambda \end{vmatrix} = -(5-\lambda)^3 = 0 \text{ for } \lambda = 5$$

(tredobbelts rod)

$$\lambda = 5 : \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad \bar{x} = r \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, r \neq 0$$

$\lambda = 5$  har alg. mult. 3 og geom. mult. 1

eks.  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ ,  $T(\bar{x}) = A\bar{x}$ ,  $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 5 \\ 0 & 0 & 2 \end{bmatrix}$

$$\begin{vmatrix} 2-\lambda & 0 & 1 \\ 0 & 2-\lambda & 5 \\ 0 & 0 & 2-\lambda \end{vmatrix} = -(2-\lambda)^3 = 0 \text{ for } \lambda = 2$$

(tredobbelts rod)

$$\lambda = 2 : \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad \bar{x} = r \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},$$

$(r,s) \neq (0,0)$

$\lambda = 2$  har alg. mult. 3 og geom. mult. 2

Similære matricer

repræsentere samme lineær operator med forskellige baser, og må derfor have samme egenværdier.

Bemerk  $A\bar{x} = \lambda\bar{x} \Leftrightarrow B^{-1}A B(B^{-1}\bar{x}) = \lambda(B^{-1}\bar{x})$ ,

dvs.  $\lambda$  egenværdi for  $A$  med egenvektor  $\bar{x}$

$\Leftrightarrow \lambda$  egenværdi for  $B^{-1}AB$  med egenvektor  $B^{-1}\bar{x}$

Koordinater mht.  $\mathcal{B}$

$T: V \rightarrow V$ ,  $\mathcal{B}$  er basis for  $V$ ,  $\dim V = n$

$\Phi: V \rightarrow \mathbb{R}^n$ ,  $\Phi(\bar{x}) = [\bar{x}]_{\mathcal{B}}$ ,  $\Phi$  er en bijektion

$\tilde{T}: \mathbb{R}^n \rightarrow \mathbb{R}^n$ ,  $\tilde{T}([\bar{x}]_{\mathcal{B}}) = [T(\bar{x})]_{\mathcal{B}}$

$$T(\bar{x}) = \lambda \bar{x} \Leftrightarrow \Phi(T(\bar{x})) = \lambda \Phi(\bar{x})$$

$$\Leftrightarrow [\tilde{T}(\bar{x})]_{\mathcal{B}} = \lambda [\bar{x}]_{\mathcal{B}}$$

$$\Leftrightarrow \tilde{T}([\bar{x}]_{\mathcal{B}}) = \lambda [\bar{x}]_{\mathcal{B}}$$

dvs.  $\lambda$  egenverdi for  $T$  med egenvektor  $\bar{x}$

$\Leftrightarrow \lambda$  egenverdi for  $\tilde{T}$  med egenvektor  $[\bar{x}]_{\mathcal{B}}$

eks.  $U: \mathbb{R}^{2 \times 2} \rightarrow \mathbb{R}^{2 \times 2}$ ,  $U(A) = A^T$

Basis for  $\mathbb{R}^{2 \times 2}$ :  $\mathcal{B} = (\bar{e}_1, \bar{e}_2, \bar{e}_3, \bar{e}_4)$

$$= ([\begin{smallmatrix} 1 & 0 \\ 0 & 0 \end{smallmatrix}], [\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}], [\begin{smallmatrix} 0 & 0 \\ 1 & 0 \end{smallmatrix}], [\begin{smallmatrix} 0 & 0 \\ 0 & 1 \end{smallmatrix}])$$

Bemerk  $U(\bar{e}_1) = \bar{e}_1$ ,  $U(\bar{e}_2) = \bar{e}_3$ ,  $U(\bar{e}_3) = \bar{e}_2$ ,  $U(\bar{e}_4) = \bar{e}_4$

$$\text{dvs. } [U]_{\mathcal{B}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 1-\lambda & 0 & 0 & 0 \\ 0 & -\lambda & 1 & 0 \\ 0 & 1 & -\lambda & 0 \\ 0 & 0 & 0 & 1-\lambda \end{vmatrix} = (1-\lambda) \begin{vmatrix} -\lambda & 1 & 0 \\ 1 & -\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix}$$

$$= (1-\lambda)^2 \begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = (\lambda-1)^2 (\lambda^2-1)$$

$$= (\lambda+1)(\lambda-1)^3 = 0 \quad \text{for } \lambda = \begin{cases} -1 \\ 1 \end{cases} \text{ (tredobbelts rot)}$$

$$\lambda=1: \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{x} = a \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + b \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} + d \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad (a, b, d) \neq (0, 0, 0)$$

$$\lambda = -1:$$

$$\begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{x} = b \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \quad b \neq 0$$

Eigenvektorer i  $\mathbb{R}^{2 \times 2}:$

$$\lambda = 1: \quad \Phi^{-1}(\tilde{x}) = \begin{bmatrix} a & b \\ b & d \end{bmatrix}, \quad (a, b, c) \neq (0, 0, 0)$$

$$\lambda = -1: \quad \Phi^{-1}(\tilde{x}) = \begin{bmatrix} 0 & b \\ -b & 0 \end{bmatrix}, \quad b \neq 0$$

ekse.  $V = \text{Sp}\{\cosh x, \sinh x\} \quad B = (\cosh x, \sinh x)$

$$T: V \rightarrow V, \quad Tf = f'$$

$$\text{Fra tidl. eks. } [T]_B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 - 1 = (\lambda + 1)(\lambda - 1) = 0 \quad \text{for } \lambda = \{-1, 1\}$$

$$\lambda = 1: \quad \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}, \quad \tilde{x} = r \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad r \neq 0$$

$$\lambda = -1: \quad \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \quad \tilde{x} = s \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad s \neq 0$$

Eigenvektorer i  $V:$

$$\lambda = 1: \quad r(\cosh x + \sinh x) = re^x, \quad r \neq 0$$

$$\lambda = -1: \quad s(\cosh x - \sinh x) = se^{-x}, \quad s \neq 0$$

(jf. eks. s. 31 i kompl.)