

Opgavesæt 1

(1)

EXERCISES

In Exercises 1–17, let  $\mathbf{u} = [-1, 3, 4]$ ,  $\mathbf{v} = [2, 1, -1]$ , and  $\mathbf{w} = [-2, -1, 3]$ . Find the indicated quantity.

1.  $\|-\mathbf{u}\|$
2.  $\|\mathbf{v}\|$
3.  $\|\mathbf{u} + \mathbf{v}\|$
4.  $\|\mathbf{v} - 2\mathbf{u}\|$
- 5.  $\|3\mathbf{u} - \mathbf{v} + 2\mathbf{w}\|$
6.  $\|\frac{4}{5}\mathbf{w}\|$
7. The unit vector parallel to  $\mathbf{u}$ , having the same direction
8. The unit vector parallel to  $\mathbf{w}$ , having the opposite direction
9.  $\mathbf{u} \cdot \mathbf{v}$
10.  $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w})$
11.  $(\mathbf{u} + \mathbf{v}) \cdot \mathbf{w}$
- 12. The angle between  $\mathbf{u}$  and  $\mathbf{v}$
13. The angle between  $\mathbf{u}$  and  $\mathbf{w}$
14. The value of  $x$  such that  $[x, -3, 5]$  is perpendicular to  $\mathbf{u}$
15. The value of  $y$  such that  $[-3, y, 10]$  is perpendicular to  $\mathbf{u}$
16. A nonzero vector perpendicular to both  $\mathbf{u}$  and  $\mathbf{v}$
17. A nonzero vector perpendicular to both  $\mathbf{u}$  and  $\mathbf{w}$

In Exercises 18–21, use properties of the dot product and norm to compute the indicated quantities mentally, without pencil or paper (or calculator).

18.  $\|[42, 14]\|$
19.  $\|[10, 20, 25, -15]\|$
20.  $[14, 21, 28] \cdot [4, 8, 20]$
21.  $[12, -36, 24] \cdot [25, 30, 10]$
22. Find the angle between  $[1, -1, 2, 3, 0, 4]$  and  $[7, 0, 1, 3, 2, 4]$  in  $\mathbb{R}^6$ .
- 23. Prove that  $(2, 0, 4)$ ,  $(4, 1, -1)$ , and  $(6, 7, 7)$  are vertices of a right triangle in  $\mathbb{R}^3$ .

24. Prove that the angle between two unit vectors  $\mathbf{u}_1$  and  $\mathbf{u}_2$  in  $\mathbb{R}^n$  is  $\arccos(\mathbf{u}_1 \cdot \mathbf{u}_2)$ .

In Exercises 25–30, classify the vectors as parallel, perpendicular, or neither. If they are parallel, state whether they have the same direction or opposite directions.

- 25.  $[-1, 4]$  and  $[8, 2]$
26.  $[-2, -1]$  and  $[5, 2]$
- 27.  $[3, 2, 1]$  and  $[-9, -6, -3]$
28.  $[2, 1, 4, -1]$  and  $[0, 1, 2, 4]$
- 29.  $[10, 4, -1, 8]$  and  $[-5, -2, 3, -4]$
30.  $[4, 1, 2, 1, 6]$  and  $[8, 2, 4, 2, 3]$
31. The distance between points  $(v_1, v_2, \dots, v_n)$  and  $(w_1, w_2, \dots, w_n)$  in  $\mathbb{R}^n$  is the norm  $\|\mathbf{v} - \mathbf{w}\|$ , where  $\mathbf{v} = [v_1, v_2, \dots, v_n]$  and  $\mathbf{w} = [w_1, w_2, \dots, w_n]$ . Why is this a reasonable definition of distance?

In Exercises 32–35, use the definition given in Exercise 31 to find the indicated distance.

32. The distance from  $(-1, 4, 2)$  to  $(0, 8, 1)$  in  $\mathbb{R}^3$
- 33. The distance from  $(2, -1, 3)$  to  $(4, 1, -2)$  in  $\mathbb{R}^3$
34. The distance from  $(3, 1, 2, 4)$  to  $(-1, 2, 1, 2)$  in  $\mathbb{R}^4$
- 35. The distance from  $(-1, 2, 1, 4, 7, -3)$  to  $(2, 1, -3, 5, 4, 5)$  in  $\mathbb{R}^6$
- 36. The captain of a barge wishes to get to a point directly across a straight river that runs from north to south. If the current flows directly downstream at 5 knots and the barge steams at 13 knots, in what direction should the captain steer the barge?
- 37. A 100-lb weight is suspended by a rope passed through an eyelet on top of the weight and making angles of  $30^\circ$  with the vertical. Find the tension (magnitude of the force vector) along the rope.

Facit : 5.  $\sqrt{478}$

12.  $103,90^\circ$

23. —

25. ortogonale

27. parallelle

29. hverken eller

33.  $\sqrt{33}$

35. 10

36.  $67,38^\circ$

37.  $\frac{100\sqrt{3}}{3}$  lb

(2) SIF afsnit 1.1 : 33, 34, 35, 36 Facit: 34. 2,798 mph  
36. gms.