

Trigonometri

Trigonometriske formler

$$\cos^2 x + \sin^2 x = 1$$

$$\tan x \cot x = 1, \quad x \neq p\frac{\pi}{2}, \quad p \in \mathbb{Z}$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}, \quad x \pm y \neq \frac{\pi}{2} + p\pi, \quad p \in \mathbb{Z}$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$$

$$\sin 2x = 2 \sin x \cos x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}, \quad x \neq \frac{\pi}{4} + p\frac{\pi}{2}, \quad p \in \mathbb{Z}$$

$$\cos x = \pm \sqrt{\frac{1 + \cos 2x}{2}}, \quad \begin{cases} + \text{ for } -\frac{\pi}{2} + p2\pi \leq x \leq \frac{\pi}{2} + p2\pi \\ - \text{ for } \frac{\pi}{2} + p2\pi \leq x \leq \frac{3\pi}{2} + p2\pi \end{cases}, \quad p \in \mathbb{Z}$$

$$\sin x = \pm \sqrt{\frac{1 - \cos 2x}{2}}, \quad \begin{cases} + \text{ for } p2\pi \leq x \leq \pi + p2\pi \\ - \text{ for } -\pi + p2\pi \leq x \leq p2\pi \end{cases}, \quad p \in \mathbb{Z}$$

$$\tan x = \frac{\sin 2x}{1 + \cos 2x} = \frac{1 - \cos 2x}{\sin 2x}, \quad x \neq \frac{\pi}{2} + p\pi, \quad p \in \mathbb{Z}$$

$$\cos x = \frac{1 - \tan^2(\frac{x}{2})}{1 + \tan^2(\frac{x}{2})}$$

$$\sin x = \frac{2 \tan(\frac{x}{2})}{1 + \tan^2(\frac{x}{2})}$$

$$\tan x = \frac{2 \tan(\frac{x}{2})}{1 - \tan^2(\frac{x}{2})}, \quad x \neq \frac{\pi}{2} + p\pi, \quad p \in \mathbb{Z}$$

$$\cos x < \frac{\sin x}{x} < 1,$$

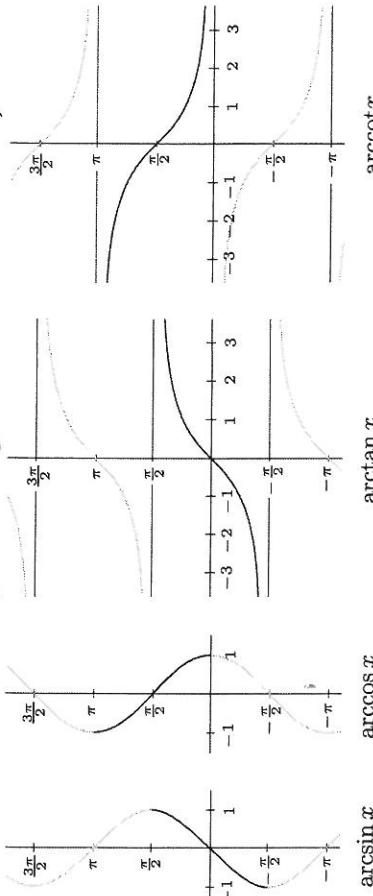
$$\sec x = \frac{1}{\cos x},$$

$$\csc x = \frac{1}{\sin x},$$

Eksakte værdier for udvalgte vinkler

Radian	sin	cos	tan	cot
0	0	1	0	∞
$\frac{\pi}{12}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$2-\sqrt{3}$	$2+\sqrt{3}$
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
$\frac{5\pi}{12}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$2+\sqrt{3}$	$2-\sqrt{3}$
$\frac{\pi}{2}$	1	0	∞	0

Cirkulære funktioner (inverse trigonometriske funktioner)



Trigonometriske grundligninger

$$\cos x = a, \quad |a| \leq 1 \quad \Leftrightarrow \quad x = \pm \arccos a + p2\pi, \quad p \in \mathbb{Z}$$

$$\sin x = a, \quad |a| \leq 1 \quad \Leftrightarrow \quad x = \begin{cases} \arcsin a + p2\pi \\ \pi - \arcsin a + p2\pi \end{cases}, \quad p \in \mathbb{Z}$$

$$\tan x = a \quad \Leftrightarrow \quad x = \arctan a + p\pi, \quad p \in \mathbb{Z}$$

$$0 < |x| < \frac{\pi}{2}$$

$$x \neq \frac{\pi}{2} + p\pi, \quad p \in \mathbb{Z}$$

$$x \neq p\pi, \quad p \in \mathbb{Z}$$

$$A \cos x + B \sin x = C, \quad 0 \leq |C| \leq \sqrt{A^2 + B^2}$$

$$\Leftrightarrow \cos(x - \varphi) = \frac{C}{\sqrt{A^2 + B^2}},$$

$$\text{hvor } \varphi \text{ er valgt så } (\cos \varphi, \sin \varphi) = \left(\frac{A}{\sqrt{A^2 + B^2}}, \frac{B}{\sqrt{A^2 + B^2}} \right)$$