Problems 10

Comments and hints are additional to those given in the book p. 617 - 618.

3.8 When H_{02} is true, cf. p. 61 l. 2,

$$\ln L(\hat{\mu}, \sigma^2) = -\frac{nd}{2}\ln(2\pi) - \frac{1}{2}\left(nd \ln \sigma^2 + \frac{1}{\sigma^2}\operatorname{tr} Q\right)$$

Differentiate w.r.t σ^2 to obtain $\hat{\sigma}^2 = \frac{\operatorname{tr} Q}{nd}$

3.9 When H_{03} is true, cf. p. 61 l. 2,

$$L(\hat{\mu}) = (2\pi)^{-\frac{nd}{2}} e^{-\frac{1}{2}\operatorname{tr} Q}$$

3.10 When H_{04} is true, cf. p. 60 formula (3.2) with (3.3) and (3.4),

$$L = (2\pi)^{-\frac{nd}{2}} (\det \Sigma_0)^{-\frac{n}{2}} \exp\left(-\frac{1}{2}\operatorname{tr}\left(\Sigma_0^{-1}\left(Q + n(\underset{\sim}{x} - \underset{\sim}{\mu_0})^T(\underset{\sim}{x} - \underset{\sim}{\mu_0})\right)\right)\right)$$

3.11 $\det C = b^{-d_0} \neq 0$

Correction to the outline solution in the book:

The columns in C do not form an orthogonal system, but they are linear independent as C is non-singular.