Problems 15

Comments and hints are additional to those given in the book p. 624.

8.9 In exercise 8.3 modified we found a confidence interval for $\underset{\sim}{a}^T \underset{\sim}{\mu}_0$, $\underset{\sim}{\mu}_0 = B^T \underset{\sim}{x}_0$. Now let $\underset{\sim}{a} := \underset{\sim}{b}$ and $\underset{\sim}{x}_0 := \underset{\sim}{a}$ to obtain a confidence interval for $\underset{\sim}{a}^T B \underset{\sim}{b}$.

Next use Bonferroni's method to find a confidence interval for m pre specified combinations of the type $\underset{\sim}{a}^T B \underset{\sim}{b}$ with an over all confidence level of at least $1-\alpha$.

Finally compare with the interval found in formula (8.63).

Results:

(i) $\underset{\sim}{a}^T B \underset{\sim}{b} = \underset{\sim}{a}^T \hat{B} \underset{\sim}{b} \pm t_{1-\frac{\alpha}{2}}(n-p) \sqrt{\underset{\sim}{a}^T (X^TX)^{-1} \underset{\sim}{a}\, \underset{\sim}{b}^T S \underset{\sim}{b}}$

(ii) $\underset{\sim}{a}^T B \underset{\sim}{b} = \underset{\sim}{a}^T \hat{B} \underset{\sim}{b} \pm t_{1-\frac{\alpha}{2m}}(n-r) \sqrt{\underset{\sim}{a}^T (X^TX)^{-1} \underset{\sim}{a}\, \underset{\sim}{b}^T S \underset{\sim}{b}}$

(iii) $\underset{\sim}{a}^T B \underset{\sim}{b} = \underset{\sim}{a}^T \hat{B} \underset{\sim}{b} \pm \sqrt{(n-r)\, q_{1-\alpha}\, \underset{\sim}{a}^T (X^TX)^{-1} \underset{\sim}{a}\, \underset{\sim}{b}^T S \underset{\sim}{b}}$

8.10 In the outline solution in the book read $|E_H|$ instead of $|E_H$.

Determine E and E_H and utilize that

$$\Lambda = \frac{\det E}{\det(E+H)} = \frac{\det E}{\det E_H} \sim U(d, p_2, n-d)$$

when H_0 is true

Find an expression for H in two ways:

(i) $H = E_H - E$

(ii) $H = Y^T (P_\Omega - P_\omega) Y$

cont.

8.10 continued

Trying to find H by use of "corollary" 1 to theorem 8.5 or by use of $P_{\omega^\perp \cap \Omega}$ leads to troublesome calculations.

8.11 In the outline solution in the book read $N_{d-1}(\underset{\sim}{0}, \frac{1}{n} C_1 \Sigma C_1^T)$ instead of $N_{d-1}(\underset{\sim}{0}, C_1 \Sigma C_1^T)$.

Consider K sets of observations:

$$\underset{\sim}{x}_i^k \sim N_d(\mu_k, \Sigma), \quad i = 1, \dots, n_k \quad k = 1, \dots, K$$

Let

$$\bar{\underset{\sim}{x}} = \frac{1}{n} \sum_{k=1}^{K} n_k \bar{\underset{\sim}{x}}^k, \quad \bar{\underset{\sim}{x}}^k = \frac{1}{n_k} \sum_{i=1}^{n_k} \underset{\sim}{x}_i^k, \quad n = \sum_{k=1}^{K} n_k$$

and

$$S_r = \frac{\sum_{i=1}^{K} (n_k - 1) S_k}{n - K}, \quad S_k = \frac{Q_k}{n_k - 1}$$

Show that

$$\left.\begin{array}{l} C_1 \bar{\underset{\sim}{x}} \sim N_{d-1}\left(\frac{1}{n} \sum_{k=1}^{K} n_k C_1 \underset{\sim}{\mu}_k, \frac{1}{n} C_1 \Sigma C_1^T\right) \\ (n-K) C_1 S_r C_1^T \sim W_{d-1}(n-K, C_1 \Sigma C_1^T) \end{array}\right\} \text{ indep.}$$

$$C_1 \bar{\underset{\sim}{x}} \sim N_{d-1}\left(\frac{1}{K} C_1 \sum_{k=1}^{K} \underset{\sim}{\mu}_k, \frac{1}{n} C_1 \Sigma C_1^T\right) \quad \text{when } H_{01} \text{ is true}$$

$$C_1 \bar{\underset{\sim}{x}} \sim N_{d-1}\left(\underset{\sim}{0}, \frac{1}{n} C_1 \Sigma C_1^T\right) \quad \text{when } H_{01} \cap H_{02} \text{ is true}$$

$$T^2 = n (C_1 \bar{\underset{\sim}{x}})^T (C_1 \Sigma C_1^T)^{-1} C_1 \bar{\underset{\sim}{x}} \sim T^2(d-1, n-K) \quad \text{when } H_{01} \cap H_{02} \text{ is true}$$

8.12 Consider the model $Y = X\underset{\sim}{\beta} + U$.

The hypothesis $H_0: A\underset{\sim}{\beta} = \underset{\sim}{0}$ can be tested when $(\underset{\sim}{a}_j^T)^T \underset{\sim}{\beta}$, $j = 1, \dots, q$, are estimable.

Assume that at least three of the x_i's are different. Then rank $X_2 = 3$, hence rank $X = 3$.