MAT 4 / MA

Problems 2

Comments and hints are addional to those given in the book p. 615.

1.7 Kronecker's delta is defined as $\delta_{rs} = \begin{cases} 1 & \text{when } r = s \\ 0 & \text{when } r \neq s \end{cases}$

1.9 Show also that

$$y^T A y \sim \sigma^2 \chi^2(r) \iff A \Sigma A = \sigma^2 A$$

This result will later be referred to as exercise 1.9 modified.

2.1 In the outline solution in the book read $N_d(\mu, I_d)$ instead of $N_n(\mu, I_n)$.

2.2 By use of moment generating functions we can state the two definitions as

<u>Definition a</u>

$$y \sim N_d(\theta, \Sigma) \iff M_y(t) = \exp\left(\underset{\sim}{t}^T \underset{\sim}{\theta} + \tfrac{1}{2} \underset{\sim}{t}^T \Sigma \underset{\sim}{t}\right)$$

<u>Definition b</u> with $\Sigma > 0$

$$y \sim N_d(\theta, \Sigma) \iff \forall \underset{\sim}{\ell} : M_{\underset{\sim}{\ell}^T \underset{\sim}{y}}(t) = \exp\left(\underset{\sim}{\ell}^T \underset{\sim}{\theta}\, t + \tfrac{1}{2} \underset{\sim}{\ell}^T \Sigma \underset{\sim}{\ell}\, t^2\right)$$

Show that definition a implies definition b, and that definition b implies definition a.

2.3 (iii) is shown in exercise 2.2

(iv) should be solved without use of moment generating functions.