MAT 4 / MA

Problems 7

Comments and hints are additional to those given in the book p. 617.

2.22 Note that $\frac{\det W}{\det \Sigma} = \det Z$, where $Z \sim W_d(m, I_d)$. Write down the factorization of $\det Z$ corresponding to formula (2.61), and utilize that the distribution of $\tilde{z}_{kk}$ is known, cf. lecture note MA 5.

2.23 Calculation of Ex^{-1}, $x \sim \chi^2(m)$:

$$Ex^{-1} = \int_0^\infty \frac{1}{x} \frac{1}{\Gamma(\frac{n}{2})} 2^{-\frac{n}{2}} x^{\frac{n-2}{2}} e^{-\frac{x}{2}} dx$$

$$= \frac{1}{2(\frac{n}{2}-1)} \int_0^\infty \frac{1}{\Gamma(\frac{n-2}{2})} 2^{-\frac{n-2}{2}} x^{\frac{(n-2)-2}{2}} e^{-\frac{x}{2}} dx$$

$$= \frac{1}{n-2}$$

2.24 Find a symmetric matrix A so that the expression in the text can be written as $X^T A X$, where $X = \begin{bmatrix} x_1^T \\ x_2^T \end{bmatrix}$

Note an additional solution $(a, c) = (1, 0)$

2.25 Use corollary 2 to theorem 2.4.