MAT 4 / MA

Problems 9

Comments and hints are additional to those given in the book p. 617.

3.4 Note that $H_0 : \underset{\sim}{\mu} = \mu \underset{\sim}{1}_d \Leftrightarrow \underset{\sim}{\mu} \in \mathcal{R}(\underset{\sim}{1}_d)$

A convenient expression for T^2 can be found in the book page 79 line 16, cf. line 13, where an expression for $\underset{\sim}{\mu}^*$ (here μ^*) is given.

Note that the expression for T^2 with μ^* inserted becomes identical with the left side of the formula in ex. 3.2.

Rewrite $\underset{\sim}{\bar{x}}^T S^{-1} \underset{\sim}{\bar{x}}$, $\underset{\sim}{\bar{x}}^T S^{-1} \underset{\sim}{1}_d$ and $\underset{\sim}{1}_d^T S^{-1} \underset{\sim}{1}_d$ as sums.

Note that $\sum_j \sum_k s^{jk} \bar{x}_j = \sum_j \sum_k s^{jk} \bar{x}_k = \frac{1}{2} \sum_j \sum_k s^{jk} (\bar{x}_j + \bar{x}_k)$.

3.5 Show that any linear combination of the q_i's is a contrast, and that any contrast is a linear combination of the q_i's.

3.6

3.7 In the text read)] instead of]).

Note that

$$\mu_2 + \rho \frac{\sigma_2}{\sigma_1} (x_1 - \mu_1) = \alpha + \beta x_1 ,$$

i.e. the test for horizontal regression line, $H_0 : \beta = 0$, is equivalent to a test for zero correlation, $H_0 : \rho = 0$.

A test statistic for H_0 is $t = \frac{\hat{\beta}}{s_{\hat{\beta}}} \sim t(n-2)$ when H_0 is true

cont.

3.7 continued

In the outline solution in the book read $\hat{\beta}_1$ instead of $\hat{\beta}$!

Intermediate results:

$$s_{\hat{\beta}}^2 = \frac{s^2}{q_{11}} = \frac{\frac{1}{n-2}\sum e_i^2}{q_{11}}$$

$$\sum_i e_i^2 = \frac{q_{11}q_{22} - q_{12}^2}{q_{11}}$$

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

The density function for $t \sim t(\nu)$ is

$$f(t) = \frac{1}{\sqrt{\nu\pi}} \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \left(1 - \frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}}, \quad -\infty < t < \infty$$