109 $X_1, \ldots, X_n$ uafh. identisk ford. kont. var.

$X_1 > X_2 > \ldots > X_{n-1} \quad \wedge \quad X_n > X_{n-1}$

$N = \min \{ n \mid X_{(n)} = X_1,\ X_{(n-1)} = X_2, \ldots, X_{(k+1)} = X_{n-k},\ X_{(k)} = X_n,$

$X_{(k-1)} = X_{n-k+1}, \ldots, X_{(2)} = X_{n-2},\ X_{(1)} = X_{n-1} \},$

$k \in \{2, \ldots, n\}$

$$P(N = n) = \sum_{k=2}^{n} \frac{1}{n!} = (n-1)\frac{1}{n!}$$

$$EN = \sum_{n=2}^{\infty} n(n-1)\frac{1}{n!} = \sum_{n=2}^{\infty} \frac{1}{(n-2)!} = \sum_{n=0}^{\infty} \frac{1}{n!} = e$$

110 $N = \min \{ n \mid X_1, \ldots, X_{n-1} \leq c,\ X_n > c \}$

$P(X_n > c) = 1 - P(X_n \leq c) = 1 - F(c)$

$N \sim g(1 - F(c))$

$$EN = \frac{1}{1 - F(c)}$$

111 X: Tidsrum ml. udbrud, $X \sim U[30; 90]$ (min.)

T: Ventetid til næste udbrud

$$ET = \frac{EX^2}{2EX} = \frac{30^2 + 30 \cdot 90 + 90^2}{3} \cdot \frac{1}{30+90} = 32{,}5 \text{ min.}$$

112 $T_1, \ldots, T_n \sim e(\lambda)$ uafh., $T = \max\{T_1, \ldots, T_n\} = T_{(n)}$

$f_T(t) = n\lambda e^{-\lambda t}(1 - e^{-\lambda t})^{n-1}$

114 $X \sim N(5,1)$, $Y \sim N(7,1)$ uafh. $M = \max\{X, Y\}$

b $F_M(t) = \Phi(t-5)\Phi(t-7)$, $F_M(6) = \Phi(1)\Phi(-1) = 0{,}1337$

a $f_M(t) = \varphi(t-5)\Phi(t-7) + \varphi(t-7)\Phi(t-5)$, $\varphi = \Phi'$

115 $X_1, \ldots, X_n \sim e(\lambda)$ uafh.

$X_{(n)} = \sum_{k=1}^{n} T_k^{(1)}$, $T_k^{(1)} \sim e((n-k+1)\lambda)$, $k = 1, \ldots, n$

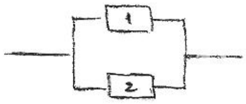
$$EX_{(n)} = \sum_{k=1}^{n} \frac{1}{(n-k+1)\lambda} = \frac{1}{\lambda}\sum_{k=1}^{n}\frac{1}{k} \approx \frac{1}{\lambda}\ln n$$

3.10

118 $X_1, \dots, X_4 \sim U[100; 200]$ uafh. , $F_{X_{(4)}}(x) = (F(x))^4$

$$F_{X_{(4)}}(x) = \left(\frac{x-100}{100}\right)^4 \geq 0{,}9 \quad \Rightarrow \quad x \geq 197{,}4$$

120



$1, 2$: $r = 1 \rightarrow r = 2$ ved kollaps af den anden

$T = \max\{T_1, T_2\}$, $T_1, T_2 \sim e(2)$

$$F_T(t) = (1 - e^{-2t})^2 \quad , \quad f_T(t) = 4e^{-2t}(1 - e^{-2t})$$

$$G_T(t) = 1 - F_T(t) = 2e^{-2t} - e^{-4t} = e^{-2t}(2 - e^{-2t})$$

$$r_T(t) = \frac{f_T(t)}{G_T(t)} = \frac{4e^{-2t}(1-e^{-2t})}{e^{-2t}(2-e^{-2t})} = \frac{4(1-e^{-2t})}{2-e^{-2t}} \rightarrow 2 \text{ for } t \rightarrow \infty$$

121 $T_1 \sim e(1)$, $T_2 \sim e(2)$ uafh.

T : levetid af tilfældigt udvalgt komponent

$$G_T(t) = P(T > t) = P(T_1 > t)\tfrac{1}{2} + P(T_2 > t)\tfrac{1}{2} = \tfrac{1}{2}(e^{-t} + e^{-2t})$$

$$f_T(t) = -G_T'(t) = \tfrac{1}{2}e^{-t} + e^{-2t}$$

$$r_T(t) = \frac{\frac{1}{2}e^{-t} + e^{-2t}}{\frac{1}{2}(e^{-t} + e^{-2t})} = \frac{e^{-t} + 2e^{-2t}}{e^{-t} + e^{-2t}} = \frac{e^t + 2}{e^t + 1}$$

122 (X_1, X_2) har simultan ford. fkt. $F(x_1, x_2)$

(X_2, X_1) – – – $F(x_2, x_1)$

$$F_{(X_{(1)}, X_{(2)})}(x_1, x_2) = F_{(X_1, X_2)}(x_1, x_2) + F_{(X_2, X_1)}(x_1, x_2)$$

$$= F(x_1, x_2) + F(x_2, x_1)$$

$$f_{(X_{(1)}, X_{(2)})}(x_1, x_2) = f(x_1, x_2) + f(x_2, x_1)$$

124 $(X_1, \dots, X_r) \sim m(n; p_1, \dots, p_r)$, $\sum_{j=1}^{r} p_r = 1$

I_{jl} : indikatorvar. for A_j i l'te elementarforsøg

$E I_{jl} = p_j$, $\operatorname{Var} I_{jl} = p_j(1 - p_j)$

fortsættes

124 fortsat

$$E[I_{jl} I_{km}] = \begin{cases} 0 & \text{for } l = m \\ E I_{jl} E I_{km} = p_j p_k & \text{for } l \neq m \end{cases} \qquad j \neq k$$

$$\mathrm{Cov}[I_{jl}, I_{km}] = \begin{cases} 0 - p_j p_k = -p_j p_k & \text{for } l = m \\ p_j p_k - p_j p_k = 0 & \text{for } l \neq m \end{cases} \qquad j \neq k$$

$$X_j = \sum_{l=1}^{n} I_{jl}, \quad E X_j = n p_j, \quad \mathrm{Var}\, X_j = n p_j (1-p_j)$$

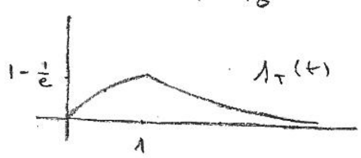
$$\begin{aligned} \mathrm{Cov}[X_j, X_k] &= \mathrm{Cov}\Big[\sum_{l=1}^{n} I_{jl}, \sum_{m=1}^{n} I_{km}\Big] \\ &= \sum_{m} \mathrm{Cov}[I_{jm}, I_{km}] + \sum_{l \neq m} \mathrm{Cov}[I_{jl}, I_{km}] \\ &= n(-p_j p_k) + n(n-1) \cdot 0 \\ &= -n p_j p_k, \quad j \neq k \end{aligned}$$

$$\begin{aligned} \rho(X_j, X_k) &= \frac{-n p_j p_k}{\sqrt{n p_j (1-p_j)}\sqrt{n p_k (1-p_k)}} \\ &= -\sqrt{\frac{p_j p_k}{(1-p_j)(1-p_k)}} \end{aligned}$$

126 $\quad W \sim e(1), \quad C \sim U[0;1]$ uafh.

$$T = W + C \qquad t = w + c \iff \begin{cases} c = t - w \\ w = t - c \end{cases}$$

$$f_T(t) = \begin{cases} \int_0^t e^{-w} \cdot 1 \, dw = \left[-e^{-w}\right]_0^t = 1 - e^{-t}, & 0 \le t \le 1 \\ \int_0^1 1 \cdot e^{-(t-c)} \, dc = e^{-t}\left[e^{c}\right]_0^1 = (e-1)e^{-t}, & 1 \le t < \infty \end{cases}$$



128 $\quad X, Y$ uafh. m. tæth. $f(x) = \exp(1-x), \quad 1 \le x < \infty$

$$Z = X + Y$$

$$\begin{aligned} f_{X+Y}(z) &= \int_1^{z-1} \exp(1-x)\exp(1-(z-x))\, dx \\ &= \exp(2-z) \int_1^{z-1} dx = (z-2)\exp(2-z), \quad 2 \le z < \infty \end{aligned}$$

129 $X \sim e(\lambda_1)$, $Y \sim e(\lambda_2)$ uavh. , $Z = X + Y$

$$f_{X+Y}(z) = \int_0^z \lambda_1 e^{-\lambda_1 x} \lambda_2 e^{-\lambda_2 (z-x)}\,dx = \lambda_1\lambda_2 e^{-\lambda_2 z}\int_0^z e^{(\lambda_2-\lambda_1)x}\,dx$$

$$= \lambda_1\lambda_2 e^{-\lambda_2 z}\left[\frac{1}{\lambda_2-\lambda_1} e^{(\lambda_2-\lambda_1)x}\right]_0^z = \frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1} e^{-\lambda_2 z}\left(e^{(\lambda_2-\lambda_1)z}-1\right)$$

$$= \frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1}\left(e^{-\lambda_1 z} - e^{-\lambda_2 z}\right) \quad , \quad 0 \le z < \infty$$

130 (X,Y) har simultan tetth. $f(x,y)$

$Z = X+Y$ $\qquad \begin{matrix} x = x \\ z = x+y \end{matrix} \qquad \begin{matrix} x = x \\ y = z-x \end{matrix} \qquad J(x,z) = \begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix} = 1$

$$f_{(X,Z)}(x,z) = f(x, z-x)\,|1| = f(x, z-x)$$

$$f_{X+Y}(z) = \int_{-\infty}^{\infty} f(x, z-x)\,dx$$

131 $f(x,y) = \frac{1}{2}(x+y)e^{-(x+y)}$, $\quad 0 \le x, y < \infty$

$$f_{X+Y}(z) = \int_0^z \frac{1}{2}(x + (z-x))e^{-(x+(z-x))}\,dx$$

$$= \frac{1}{2} z e^{-z}\int_0^z dx = \frac{1}{2} z^2 e^{-z} \quad , \quad 0 \le z < \infty$$