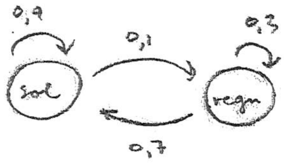
7.2

1 a



$$P = \begin{bmatrix} 0{,}9 & 0{,}1 \\ 0{,}7 & 0{,}3 \end{bmatrix}.$$

b $p = 0{,}9^2 + 0{,}1 \cdot 0{,}7 = 0{,}88$

c $p = 0{,}9^2 = 0{,}81$

2 Betingelse: $0{,}7 + 3p = 1 \Rightarrow p = 0{,}1$

1. række i P: $k(0{,}7 + 0{,}1) = 1 \Rightarrow k = \frac{10}{8} \Rightarrow p_{ss} = \frac{7}{8} \wedge p_{sr} = \frac{1}{8}$

2. række i P: $k(0{,}1 + 0{,}1) = 1 \Rightarrow k = \frac{10}{2} \Rightarrow p_{rs} = \frac{1}{2} \wedge p_{rr} = \frac{1}{2}$

$$P = \begin{bmatrix} \frac{7}{8} & \frac{1}{8} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

3 $p_{dd} = 0{,}3 \Rightarrow p_{dn} = 0{,}7$

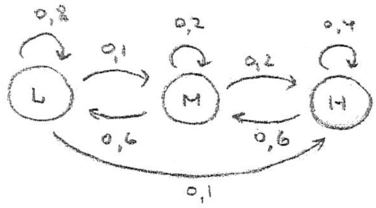
$p_{nd} = 0{,}01 \Rightarrow p_{nn} = 0{,}99$

$$P = \begin{bmatrix} 0{,}3 & 0{,}7 \\ 0{,}01 & 0{,}99 \end{bmatrix}$$

$$\lim_{n \to \infty} P^n = \frac{1}{0{,}71} \begin{bmatrix} 0{,}01 & 0{,}7 \\ 0{,}01 & 0{,}7 \end{bmatrix}$$ jf. eks. 7.2.6 s. 413

Andel af defekte: $\frac{0{,}01}{0{,}71} = \frac{1}{71} = 0{,}0141$

4 a



$$P = \begin{bmatrix} 0{,}8 & 0{,}1 & 0{,}1 \\ 0{,}6 & 0{,}2 & 0{,}2 \\ 0 & 0{,}6 & 0{,}4 \end{bmatrix}$$

b X: antal år i L ved start i L

$X + 1 \sim g(1 - 0{,}8) = g(0{,}2)$

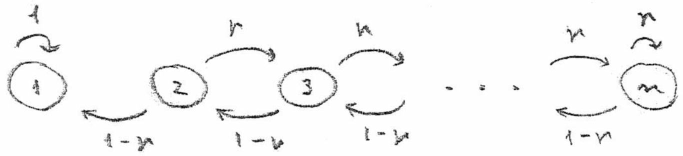
$EX = \frac{1}{0{,}2} - 1 = 5 - 1 = 4$

6 Tilstand i er transient og $i \to j$

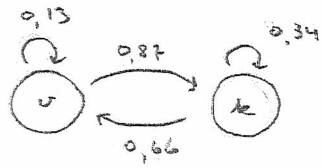
Tilstand j kan være rekurrent, hvis $j \not\to i$ og S er endelig, jf. korollar 7.2.2 s. 415

7.2

7 $S = \{1, \dots, n\}$, random walk med én reflekterende barriere og én absorberende barriere :



9 a



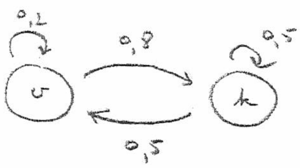
$$P = \begin{bmatrix} 0,13 & 0,87 \\ 0,66 & 0,34 \end{bmatrix}$$

b $\sum_{k=1}^{2} p_{1k} p_{k2} = 0,13^2 + 0,87 \cdot 0,66 = 0,5911$

c andel af vokaler : $\dfrac{0,66}{0,66+0,87} = 0,4314$

andel af konsonanter : $\dfrac{0,87}{0,66+0,87} = 0,5686$

10



a k $\left(\dfrac{0,8}{1,3} > \dfrac{0,5}{1,3} \right)$

b k $(0,8 + 0,5 > 0,2 + 0,5)$

c k, k, k, k, k (jf. spm. a)

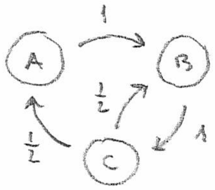
d k, v, k, v, k $(0,5 \cdot 0,8 > 0,5^2)$

<u>SLB</u>

3.1

$$P = \begin{bmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & 0 & \frac{2}{3} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

3.5



i $$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

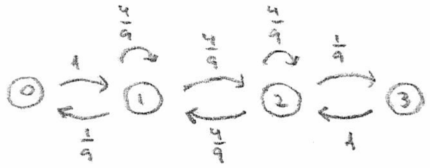
ii $$P^2 = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}, \quad P^4 = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{bmatrix},$$

$$P^5 = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{8} & \frac{3}{8} & \frac{1}{2} \end{bmatrix}, \quad \forall (i,j) : r_{ij}^{(5)} > 0$$

$\Rightarrow$ Markovkaden er regulær

3.9

i



$$P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{1}{9} & \frac{4}{9} & \frac{4}{9} & 0 \\ 0 & \frac{4}{9} & \frac{4}{9} & \frac{1}{9} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

ii $$P^2 = \begin{bmatrix} \frac{1}{9} & \frac{4}{9} & \frac{4}{9} & 0 \\ \frac{4}{81} & \frac{41}{81} & \frac{32}{81} & \frac{4}{81} \\ \frac{4}{81} & \frac{32}{81} & \frac{41}{81} & \frac{4}{81} \\ 0 & \frac{4}{9} & \frac{4}{9} & \frac{1}{9} \end{bmatrix}, \quad P^3 = \begin{bmatrix} \frac{4}{81} & \frac{41}{81} & \frac{32}{81} & \frac{4}{81} \\ \frac{41}{729} & \frac{328}{729} & \frac{328}{729} & \frac{32}{729} \\ \frac{32}{729} & \frac{328}{729} & \frac{328}{729} & \frac{41}{729} \\ \frac{4}{81} & \frac{32}{81} & \frac{41}{81} & \frac{4}{81} \end{bmatrix}$$

$\forall (i,j) : r_{ij}^{(3)} > 0 \Rightarrow$ Markovkaden er regulær

iii $r_{00}^{(1)} = 0$

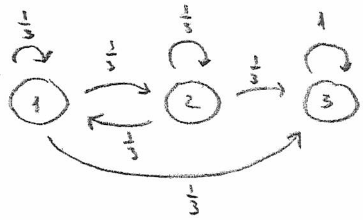
$r_{00}^{(2)} = \frac{1}{9}$

$r_{00}^{(3)} = \frac{4}{81}$

$r_{00}^{(4)} = r_{10}^{(3)} = \frac{41}{729}$

SLB

4.2 $$P = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 1 \end{bmatrix}$$

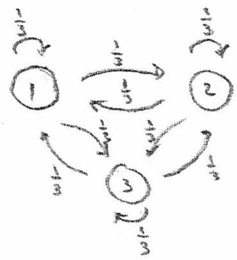


i $$f_{11}^{(n)} = \left(\frac{1}{3}\right)^n, \quad n = 1, 2, \ldots$$

ii Tilstand 1 er transient

$$f_{11} = \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n = \frac{1}{3} \frac{1}{1-\frac{1}{3}} = \frac{1}{2}$$

4.3 $$P = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$



i $$f_{11}^{(n)} = \frac{1}{3}\left(\frac{2}{3}\right)^{n-1}$$

ii Tilstand 1 er rekurrent (persistent)

$$\left(f_{11} = \sum_{n=1}^{\infty} \frac{1}{3}\left(\frac{2}{3}\right)^{n-1} = \frac{1}{3} \frac{1}{1-\frac{2}{3}} = 1 \right)$$

iii $$T_{11} \sim g\left(\frac{1}{3}\right)$$

$$\mu_{11} = E[T_{11}] = \frac{1}{\frac{1}{3}} = 3$$

4.7 Tilstand i er transient.

f_{ii} er sands. for genbesøg i tilstand i.

N_{ii} er antal genbesøg i tilstand i.

$$N_{ii} + 1 \sim g(1 - f_{ii})$$

$$E[N_{ii}] = \frac{1}{1-f_{ii}} - 1 = \frac{f_{ii}}{1-f_{ii}}$$