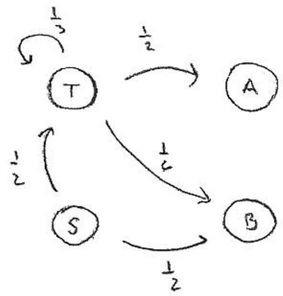
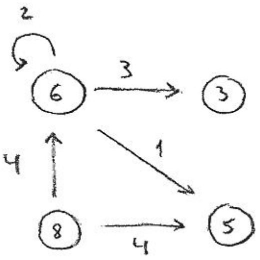
SFK

1.7



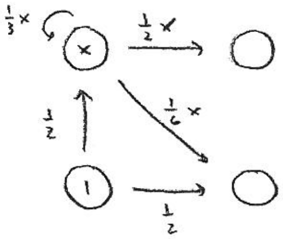


$P(A) = \frac{3}{8}$

$P(B) = \frac{5}{8}$

gns. tid : $\frac{8+6}{3+5} = \frac{7}{4} = 1{,}75$

gennemstrømningsmetoden :



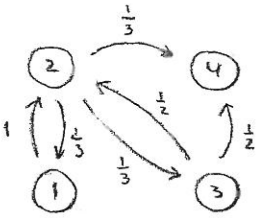
$\frac{1}{2}\,\frac{3}{4} = \frac{3}{8}$

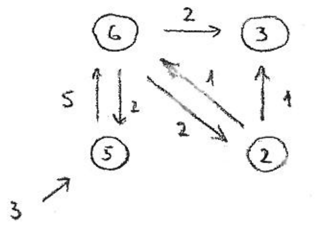
$\frac{1}{2} + \frac{1}{6}\,\frac{3}{4} = \frac{5}{8}$

gns. tid : $1 + \frac{3}{4} = \frac{7}{4} = 1{,}75$

$x = \frac{1}{2} + \frac{1}{3}x \implies x = \frac{3}{4}$

1.8

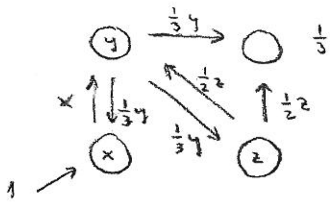




gns. tid :

$\frac{5+6+2}{3} = \frac{13}{3} \approx 4{,}33$

gennemstrømningsmetoden :



$\frac{1}{3}z + \frac{1}{2}\,\frac{2}{3} = 1$

$$\left.\begin{aligned} x &= 1 + \tfrac{1}{3}y \\ y &= x + \tfrac{1}{2}z \\ z &= \tfrac{1}{3}y \end{aligned}\right\}\; y = x + \tfrac{1}{6}y,\quad x = \tfrac{5}{6}y \;\Big\}\; \tfrac{5}{6}y = 1 + \tfrac{1}{3}y,\quad y = 2$$

$x = \frac{5}{3}$

$z = \frac{2}{3}$

gns. tid : $x + y + z = \frac{5}{3} + 2 + \frac{2}{3} = \frac{13}{3} \approx 4{,}33$

matrixmetoden :

$$Q = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{2} & 0 \end{bmatrix},\quad E - Q = \begin{bmatrix} 1 & -1 & 0 \\ -\frac{1}{3} & 1 & -\frac{1}{3} \\ 0 & -\frac{1}{2} & 1 \end{bmatrix},$$

$\det(E-Q)$
$= \frac{5}{6} + \left(-\frac{1}{3}\right) = \frac{1}{2}$

fortsættes

SFK

1.8 fortsat

$$(E-Q)^{-1} = \frac{1}{\frac{1}{2}} \begin{bmatrix} \frac{5}{6} & \frac{1}{3} & \frac{1}{6} \\ 1 & 1 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & \frac{2}{3} \end{bmatrix}^T = 2 \begin{bmatrix} \frac{5}{6} & 1 & \frac{1}{3} \\ \frac{1}{3} & 1 & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{2} & \frac{2}{3} \end{bmatrix} , \quad N_1 = 2\left(\frac{5}{6} + 1 + \frac{1}{3}\right) = \frac{13}{3} \approx 4{,}33$$

4.3

$$P = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix} , \quad \text{i} \quad E = [1] , \quad R = \begin{bmatrix} \frac{1}{3} \\ 0 \end{bmatrix} , \quad Q = \begin{bmatrix} \frac{2}{3} & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

ii

$$Q^2 = \begin{bmatrix} \frac{2}{3} & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{2}{3} & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{4}{9} & 0 \\ \frac{7}{12} & \frac{1}{4} \end{bmatrix}$$

$$Q^3 = \begin{bmatrix} \frac{2}{3} & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{4}{9} & 0 \\ \frac{7}{12} & \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{8}{27} & 0 \\ \frac{37}{72} & \frac{1}{8} \end{bmatrix}$$

$$Q^4 = \begin{bmatrix} \frac{2}{3} & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{8}{27} & 0 \\ \frac{37}{72} & \frac{1}{8} \end{bmatrix} = \begin{bmatrix} \frac{16}{81} & 0 \\ \frac{175}{432} & \frac{1}{16} \end{bmatrix}$$

$$(E+Q)R = \begin{bmatrix} \frac{5}{3} & 0 \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{5}{9} \\ \frac{1}{6} \end{bmatrix}$$

$$(E+Q+Q^2)R = \begin{bmatrix} \frac{19}{9} & 0 \\ \frac{13}{12} & \frac{7}{4} \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{19}{27} \\ \frac{13}{36} \end{bmatrix}$$

$$(E+Q+Q^2+Q^3)R = \begin{bmatrix} \frac{65}{27} & 0 \\ \frac{115}{72} & \frac{15}{8} \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{65}{81} \\ \frac{115}{216} \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 1 & 0 & 0 \\ \frac{5}{9} & \frac{4}{9} & 0 \\ \frac{1}{6} & \frac{7}{12} & \frac{1}{4} \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 1 & 0 & 0 \\ \frac{19}{27} & \frac{8}{27} & 0 \\ \frac{13}{36} & \frac{37}{72} & \frac{1}{8} \end{bmatrix}$$

$$P^4 = \begin{bmatrix} 1 & 0 & 0 \\ \frac{65}{81} & \frac{16}{81} & 0 \\ \frac{115}{216} & \frac{175}{432} & \frac{1}{16} \end{bmatrix}$$

iii

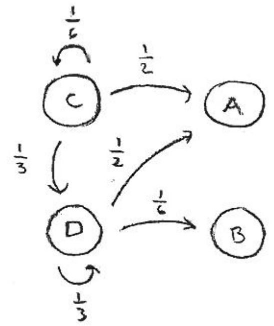
$$\lim_{n \to \infty} P^n = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} , \quad \text{idet} \quad \lim_{n \to \infty} Q^n = 0$$

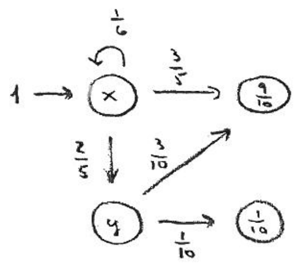
kontrolregninger : $E - Q = \begin{bmatrix} \frac{1}{3} & 0 \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$; $\det(E-Q) = \frac{1}{6}$

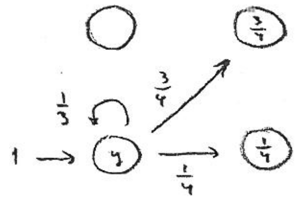
$$(E-Q)^{-1} = \frac{1}{\frac{1}{6}} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{3} \end{bmatrix}^T = \begin{bmatrix} 3 & 0 \\ 3 & 2 \end{bmatrix} , \quad B = (E-Q)^{-1} R = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

SFK

4.4







$x = 1 + \frac{1}{6}x \Rightarrow x = \frac{6}{5}$ $y = 1 + \frac{1}{3}y \Rightarrow y = \frac{3}{2}$

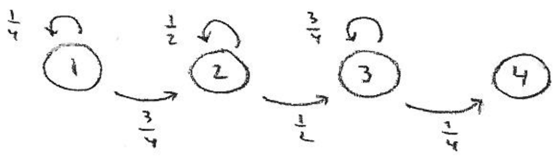
$y = \frac{2}{5} + \frac{1}{3}y \Rightarrow y = \frac{3}{5}$

$$N = \begin{bmatrix} \frac{6}{5} & \frac{3}{5} \\ 0 & \frac{3}{2} \end{bmatrix}, \quad N_C = \frac{6}{5} + \frac{3}{5} = \frac{9}{5}$$

$$N_D = 0 + \frac{3}{2} = \frac{3}{2}$$

kontrolregning: $B = NR = \begin{bmatrix} \frac{6}{5} & \frac{3}{5} \\ 0 & \frac{3}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{6} \end{bmatrix} = \begin{bmatrix} \frac{9}{10} & \frac{1}{10} \\ \frac{3}{4} & \frac{1}{4} \end{bmatrix}$

4.5



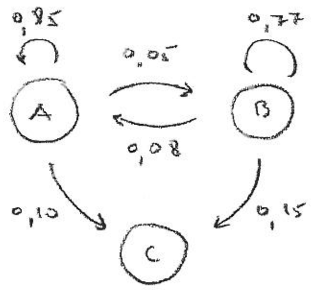
forskellige farver

$1 \longrightarrow \frac{4}{3} \xrightarrow{1} 2 \xrightarrow{1} 4 \xrightarrow{1} 1$

$N = \frac{4}{3} + 2 + 4 = \frac{22}{3}$

$\approx 7{,}33$

4.6



$$Q = \begin{bmatrix} 0{,}85 & 0{,}05 \\ 0{,}08 & 0{,}77 \end{bmatrix}$$

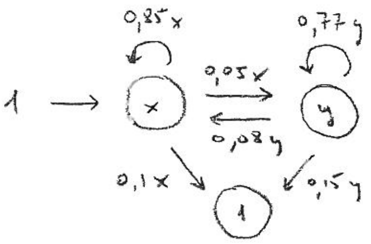
$$E - Q = \begin{bmatrix} 0{,}15 & -0{,}05 \\ -0{,}08 & 0{,}23 \end{bmatrix}$$

$$\det(E - Q) = 0{,}0305 = \frac{61}{2000}$$

$$N = \frac{2000}{61} \begin{bmatrix} 0{,}23 & 0{,}08 \\ 0{,}05 & 0{,}15 \end{bmatrix}^T = \frac{1}{61} \begin{bmatrix} 460 & 100 \\ 160 & 300 \end{bmatrix}$$

$$N_A = \frac{1}{61}(460 + 100) = \frac{560}{61} \approx 9{,}18 \text{ mdr.}$$

gennemstrømningsmetoden:



$x = 1 + \frac{85}{100}x + \frac{8}{100}y$

$y = \frac{5}{100}x + \frac{77}{100}y, \quad y = \frac{5}{23}x$

$\frac{3}{20}x = 1 + \frac{4}{230}x, \quad x = \frac{460}{61}, \quad y = \frac{100}{61}$

$N_A = x + y = \frac{560}{61} = 9{,}18$ mdr.

SFK

4.7 1) $N = (E-Q)^{-1} \Rightarrow N^{-1} = E - Q \Rightarrow Q = E - N^{-1}$

2) $NQ = NE - NN^{-1} = N - E$

4.8 Træfsandsynligheder $A : \frac{2}{3}$ $B : \frac{1}{2}$ $C : \frac{1}{3}$

1) AB ikke mulig, da ingen skyder på C før A eller B er elimineret, alle andre kombinationer er mulige, dvs.

$T = \{ABC, AC, BC, A, B, C, \emptyset\}$

2) $P(ABC \to ABC) = \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{9}$

$P(ABC \to AC) = \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{2}{3} = \frac{2}{9}$

$P(ABC \to BC) = \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{2}{3} + \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{3} = \frac{2}{9}$

$P(ABC \to C) = \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{2}{3} + \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{3} + \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{3} + \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{3} = \frac{4}{9}$

$P(ABC \to \emptyset) = 0$

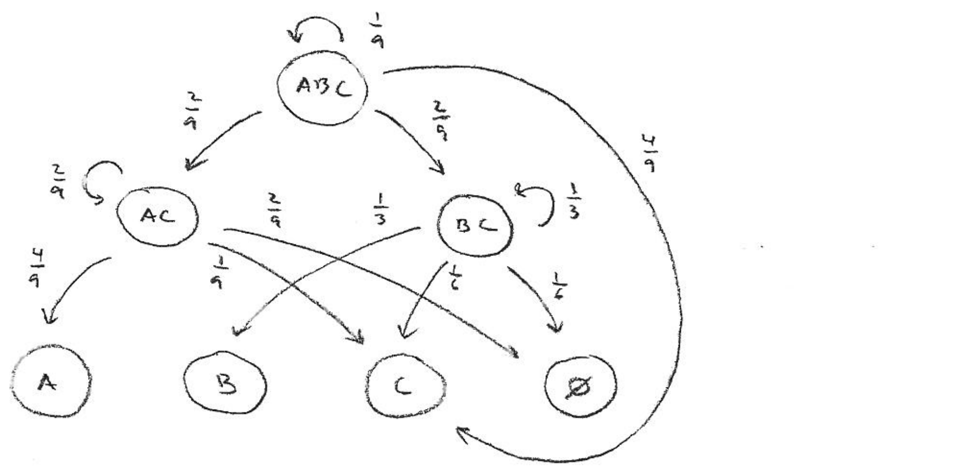
$P(AC \to AC) = \frac{1}{3} \cdot \frac{2}{3} = \frac{2}{9}$ $P(BC \to BC) = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$

$P(AC \to A) = \frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9}$ $P(BC \to B) = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$

$P(AC \to C) = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$ $P(BC \to C) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$

$P(AC \to \emptyset) = \frac{2}{3} \cdot \frac{1}{3} = \frac{2}{9}$ $P(BC \to \emptyset) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$

$$P = \begin{array}{c} \begin{array}{ccccccc} \emptyset & A & B & C & AC & BC & ABC \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{2}{9} & \frac{4}{9} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{4}{9} & \frac{2}{9} & \frac{2}{9} & \frac{1}{9} \end{bmatrix} \end{array} \begin{array}{c} \emptyset \\ A \\ B \\ C \\ AC \\ BC \\ ABC \end{array}$$

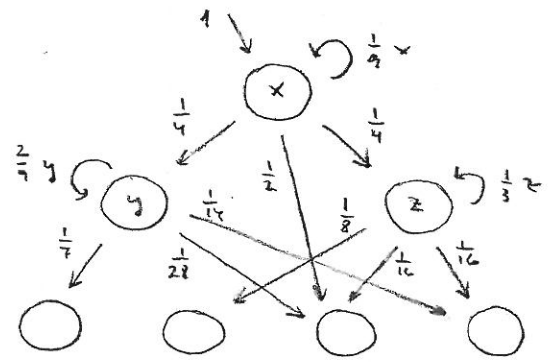


fortsættes

SFK

4.8 fortsat

3)



$$x = 1 + \frac{1}{9}x \Rightarrow x = \frac{9}{8}$$

$$y = \frac{1}{4} + \frac{2}{9}y \Rightarrow y = \frac{9}{28}$$

$$z = \frac{1}{4} + \frac{1}{3}z \Rightarrow z = \frac{3}{8}$$

$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{28} + \frac{1}{2} + \frac{1}{16}$	$\frac{1}{14} + \frac{1}{16}$
$= \frac{16}{112}$	$= \frac{14}{112}$	$= \frac{67}{112}$	$= \frac{15}{112}$

4) Forventet varighed af tankeslaget:

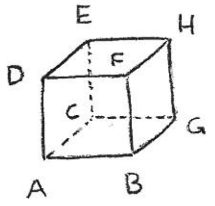
$$x + y + z = \frac{9}{8} + \frac{9}{28} + \frac{3}{8} = \frac{51}{28} \approx 1,82$$

Forventet antal affyrede skud:

$$3 \cdot \frac{9}{8} + 2\left(\frac{9}{28} + \frac{3}{8}\right) = \frac{267}{56} \approx 4,77$$

SLB

7.2



Random walk på hjørnerne af terning

Antag, at A og B er absorberende tilstande

i Sandsynlighed for absorbtion i A

- ved start i C eller D:

$$r_1 = \frac{1}{3}1 + \frac{1}{3}r_2 + \frac{1}{3}r_3 \iff 3r_1 - r_2 - r_3 = 1$$

- ved start i E:

$$r_2 = \frac{2}{3}r_1 + \frac{1}{3}r_4 \iff 2r_1 - 3r_2 + r_4 = 0$$

- ved start i F eller G:

$$r_3 = \frac{1}{3}0 + \frac{1}{3}r_1 + \frac{1}{3}r_4 \iff r_1 - 3r_3 + r_4 = 0$$

- ved start i H:

$$r_4 = \frac{1}{3}r_2 + \frac{2}{3}r_3 \iff r_2 + 2r_3 - 3r_4 = 0$$

$$\left[\begin{array}{cccc|c} 3 & -1 & -1 & 0 & 1 \\ 2 & -3 & 0 & 1 & 0 \\ 1 & 0 & -3 & 1 & 0 \\ 0 & 1 & 2 & -3 & 0 \end{array}\right] \sim \cdots \sim \left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 2 & -4 & -1 \\ 0 & 0 & 0 & 7 & 3 \end{array}\right]$$

$$(r_1, r_2, r_3, r_4) = \left(\frac{9}{14}, \frac{4}{7}, \frac{5}{14}, \frac{3}{7}\right)$$

ii Forventet tid til absorbtion i A eller B

- ved start i C, D, F eller G:

$$\mu_1 = 1 + \frac{2}{3}\mu_1 + \frac{1}{3}\mu_2 \iff \mu_1 = \frac{3}{2} + \frac{1}{2}\mu_2$$

- ved start i E eller H:

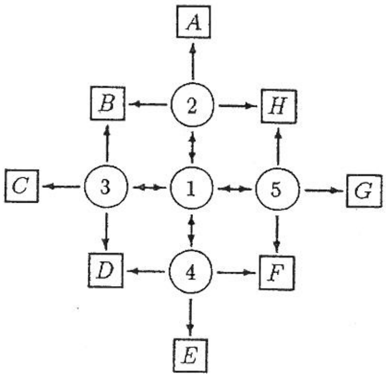
$$\mu_2 = 1 + \frac{2}{3}\mu_1 + \frac{1}{3}\mu_2 \iff \mu_2 = \frac{3}{2} + \mu_1$$

$$\mu_1 = \frac{3}{2} + \frac{1}{2}\left(\frac{3}{2} + \mu_1\right) \Rightarrow \mu_1 = \frac{9}{2} \Rightarrow \mu_2 = 6$$

$$(\mu_1, \mu_2) = \left(\frac{9}{2}, 6\right)$$

SLB

7.7



Random walk på gitter

□ markerer absorberende tilstande

○ markerer transiente tilstande

i Absorbtionssandsynligheder :

$$r_{1A} = \frac{1}{4} r_{2A} + \frac{3}{4} r_{3A} \qquad r_{1B} = \frac{1}{2} r_{2B} + \frac{1}{2} r_{4B}$$

$$r_{2A} = \frac{1}{4} + \frac{1}{4} r_{2A} \qquad r_{2B} = \frac{1}{4} + \frac{1}{4} r_{1B}$$

$$r_{3A} = \frac{1}{4} r_{1A} \qquad r_{4B} = \frac{1}{4} r_{1B}$$

$$r_{1A} = \frac{1}{4}\left(\frac{1}{4} + \frac{1}{4} r_{1A}\right) + \frac{3}{4}\frac{1}{4} r_{1A} \Rightarrow r_{1A} = \frac{1}{12}$$

$$r_{1B} = \frac{1}{2}\left(\frac{1}{4} + \frac{1}{4} r_{1B}\right) + \frac{1}{2}\frac{1}{4} r_{1B} \Rightarrow r_{1B} = \frac{1}{6}$$

$$(r_{1A}, r_{2A}, r_{3A}) = \left(\frac{1}{12}, \frac{13}{48}, \frac{1}{48}\right)$$

$$(r_{1B}, r_{2B}, r_{4B}) = \left(\frac{1}{6}, \frac{7}{24}, \frac{1}{24}\right)$$

ii Forventet tid til absorbtion

- ved start i tilstand 1 : $\mu_1 = 1 + \mu_2$
- ved start i tilstand 2 : $\mu_2 = 1 + \frac{1}{4}\mu_1$

$$\mu_1 = 1 + 1 + \frac{1}{4}\mu_1 \Rightarrow \mu_1 = \frac{8}{3} \Rightarrow \mu_2 = \frac{5}{3}$$

$$(\mu_1, \mu_2) = \left(\frac{8}{3}, \frac{5}{3}\right)$$

Øvrige absorbtionssandsynligheder og forventede tider fås ved symmetri-betragtninger.