1.2

1 a $\{3, \dots, 18\}$

b $[0;1] \times [0;1]$

c $\{m, k\} \times \{0, 1, \dots\}$

d $\{(m,n)\}$, $1 \le m < n \le 10$

e $[0;1]$

3 a $(A \cap B^c \cap C^c) \cup (A^c \cap B \cap C^c) \cup (A^c \cap B^c \cap C)$

b $A^c \cap B^c \cap C^c$

c $A \cup B \cup C$

d $A \cap B \cap C$

1.3

5 $P(A) = P(B) = \frac{1}{2}$, $P(A \cap B) = p$

a $P(A \cap B^c) = P(A) - P(A \cap B) = \frac{1}{2} - p$

b $P(A \cap B^c) + P(A^c \cap B) = 1 - 2p$

c $P(A^c \cap B^c) = P((A \cup B)^c) = 1 - P(A \cup B)$

$= 1 - (1 - p) = p$

alt. $P(A^c \cap B^c) = P(A \cap B)$ pga. sym.

6 Jo større sands. for regn begge dage , des mindre sands. for regn kun én af dagene.

7 a $0{,}8 \neq 0{,}7 + 0{,}5 - 0{,}4$

b $0{,}9 = 0{,}7 + 0{,}6 - 0{,}4$ ok

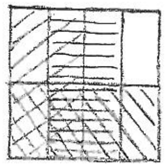
c $0{,}8 > 0{,}5$

d $0{,}9 \neq 0{,}7 + 0{,}6 - 0{,}5$

1.3

9

$$P(A) = P(B) = P(C) = \frac{1}{2}$$



$$P(A \cap B) = P(A \cap C) = P(B \cap C) = \frac{1}{4}$$

$$P(A \cap B \cap C) = \frac{1}{8}$$

a $\quad 3P(A \cap B^c \cap C^c) = 3P(A \cap B \cap C) = \frac{3}{8}$ (sym. udnyttet)

b $\quad 1 - P(A \cup B \cup C) = 1 - \left(3\frac{1}{2} - 3 \cdot \frac{1}{4} + \frac{1}{8}\right) = 1 - \frac{7}{8} = \frac{1}{8}$

c $\quad P(A \cup B \cup C) = \frac{7}{8}$

d $\quad P(A \cap B \cap C) = \frac{1}{8}$

e $\quad 1 - 3P(A \cap B^c \cap C^c) - 2P(A \cap B \cap C) = 1 - \frac{3}{8} - \frac{2}{8} = \frac{3}{8}$

10 a $\quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow \begin{cases} P(A \cup B) \geq P(A) + B(B) - 1 \\ P(A \cup B) \leq P(A) + P(B) \end{cases}$$

b $\quad$ induktion

i $\quad$ gælder for $n=2$, jf. spm. a

ii $\quad$ antag, at

$$\sum_{k=1}^{n-1} P(A_k) - (n-2) \leq P\left(\bigcup_{k=1}^{n-1} A_k\right) \leq \sum_{k=1}^{n-1} P(A_k)$$

spm. a og induktionsantagelsen udnyttes:

$$P\left(\bigcup_{k=1}^{n} A_k\right) = P\left(\left(\bigcup_{k=1}^{n-1} A_k\right) \cup A_n\right)$$

$$\begin{cases} \geq \left(\sum_{k=1}^{n-1} P(A_k) - (n-2)\right) + P(A_n) - 1 \\ \leq \sum_{k=1}^{n-1} P(A_k) + P(A_n) \end{cases}$$

$$\Downarrow$$

$$\sum_{k=1}^{n} P(A_k) - (n-1) \leq P\left(\bigcup_{k=1}^{n} A_k\right) \leq \sum_{k=1}^{n} P(A_k)$$

11 $\quad P(W_i \geq 10) = 0{,}01 \quad \Rightarrow \quad P\left(\bigcup_{i=1}^{10} (W_i \geq 10)\right) \leq 10 \cdot 0{,}01 = 0{,}1$

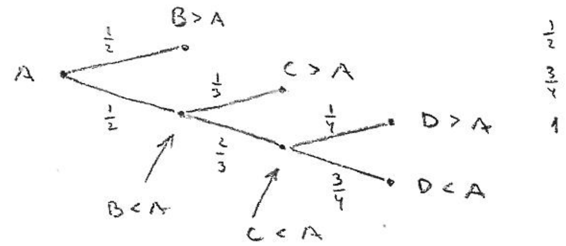
$i = 1, \ldots, 10$

1.4

16 a $26^5 \, 10^2 = 1.188.137.600$

b $(26)_5 \, (10)_2 = 710.484.000$

19 b



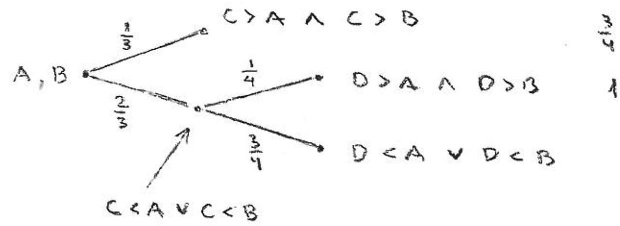
a P(største beløb) $= \frac{1}{4}$

$$P(C>A \mid B<A) = \frac{P(B<A<C)}{P(B<A)} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$$

$$P(D>A \mid B<A \wedge C<A) = \frac{P\left(\left\{\begin{smallmatrix} B \\ C \end{smallmatrix}\right\} < A < D\right)}{P(B<A \wedge C<A)} = \frac{\frac{2}{24}}{\frac{1}{2} \cdot \frac{2}{3}} = \frac{1}{4}$$

$$P(\text{største beløb}) = \frac{1}{2}\left(\frac{1}{2} + \frac{1}{3} \cdot \frac{3}{4} + \frac{2}{3} \cdot \frac{1}{4} \cdot 1\right) = \frac{11}{24}$$

c



$$P(\text{største beløb}) = \frac{1}{3} \cdot \frac{3}{4} + \frac{2}{3} \cdot \frac{1}{4} \cdot 1 = \frac{5}{12}.$$

20 a i $\binom{4}{2} = 6$ ii $2\binom{4}{2} = 12$

b i $\frac{4!}{2!\,2!} = 6$ ii $\frac{4!}{1!\,2!\,1!} = 12$

21 a $\binom{n+1}{k+1} = \binom{n}{k+1} + \binom{n}{k}$

antal komb. uden et givet element + antal komb. m. dette

b $k\binom{n}{k} = n\binom{n-1}{k-1}$, idet $\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1}$

c $\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2$, idet $\binom{2n}{n} = \binom{n+n}{n} = \sum_{k=0}^{n} \binom{n}{k}\binom{n}{n-k}$

$$= \sum_{k=0}^{n} \binom{n}{k}^2$$

d $\sum_{k=0}^{n} \binom{n}{k} = 2^n$

antal forsk. delmgd.er af en n-mgd. (∅ medregnet)

1.5

33 $P(A|B) + P(A|B^c) = 1$???

Modeks., betragt $A \subset B$ (ægte delhændelse), hvor $P(A) > 0$ og $P(B) < 1$

$$\left.\begin{array}{l} P(A|B) = \dfrac{P(A\cap B)}{P(B)} = \dfrac{P(A)}{P(B)} = p < 1 \\ P(A|B^c) = \dfrac{P(A\cap B^c)}{P(B^c)} = \dfrac{P(\emptyset)}{P(B^c)} = 0 \end{array}\right\} \quad p + 0 < 1$$

34 A, B disjunkte

$$P(A|A\cup B) = \frac{P(A\cap(A\cup B))}{P(A\cup B)} = \frac{P((A\cap A)\cup(A\cap B))}{P(A)+P(B)}$$

$$= \frac{P(A\cup\emptyset)}{P(A)+P(B)} = \frac{P(A)}{P(A)+P(B)}$$

35 i $P(A\cap B\cap C) = P(A\cap(B\cap C)) = P(A|B\cap C)\,P(B\cap C)$

$$= P(A|B\cap C)\,P(B|C)\,P(C)$$

ii $P(A_1\cap A_2\cap\ldots\cap A_n)$

$$= P(A_1|A_2\cap\ldots\cap A_n)\,P(A_2|A_3\cap\ldots\cap A_n)\ldots$$

$$\ldots P(A_{n-1}|A_n)\,P(A_n)$$

37 i $P(\emptyset\cap A) = P(\emptyset) = 0 = 0\,P(A) = P(\emptyset)P(A)$

ii $P(S\cap A) = P(A) = 1\,P(A) = P(S)\,P(A)$

39 $P(A) = p^3 + 3p^2(1-p) = p^2(3-2p)$ $\quad P(B) = p^3 + (1-p)^3 = 3p^2 - 3p + 1$ $\quad P(A\cap B) = p^3$

$$P(A)P(B) = p^2(9p^2 - 9p + 3 - 6p^3 + 6p^2 - 2p) = p^2(-6p^3 + 15p^2 - 11p + 3)$$

$$P(A\cap B) = P(A)P(B) \iff p^3 = p^2(-6p^3 + 15p^2 - 11p + 3)$$

$$\iff p^2(6p^3 - 15p^2 + 12p - 3) = 0$$

$$\iff 3p^2(p-1)(2p^2 - 3p + 1) = 0$$

$$\iff 6p^2(p-1)(p-\tfrac{1}{2})(p-1) = 0$$

$$\iff p = 0 \lor p = \tfrac{1}{2} \lor p = 1$$

1.5

43 $P(A_k) = \frac{1}{6}$, $k = 1, \dots, 6$

m	2	3	4	5	6	7	8	9	10	11	12
$P(B_m)$	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

$P(A_k \cap B_m) = \frac{1}{36}$, $m = k+1, \dots, k+6$, $k = 1, \dots, 6$

$P(A_k)\,P(B_7) = P(A_k \cap B_7)$, $k = 1, \dots, 6$

dvs. $m = 7$

59 i $P = (1 - (1-p)^2)^2$

ii $P = 1 - (1-p^2)^2$