1.5

45 $P(A\cap(B\cap C)) = P(A\cap B\cap C) = P(A)P(B)P(C)$

$= P(A)P(B\cap C)$

$P(A\cap(B\cup C)) = P((A\cap B)\cup(A\cap C))$

$= P(A\cap B) + P(A\cap C) - P((A\cap B)\cap(A\cap C))$

$= P(A)P(B) + P(A)P(C) - P(A)P(B)P(C)$

$= P(A)(P(B) + P(C) - P(A\cap B))$

$= P(A)P(B\cup C)$

47 a $P(\text{sammenfald}) = \sum_{k=1}^{365} p_k^2$, $\sum_{k=1}^{365} p_k = 1$ (bibetingelse)

$\Leftrightarrow \sum_{k=1}^{365} p_k - 1 = 0$

b Lagranges multiplikatormetode:

$$\frac{\partial}{\partial p_k}\left(\sum_{k=1}^{365} p_k^2 - \lambda\left(\sum_{k=1}^{365} p_k - 1\right)\right) = 2p_k - \lambda = 0$$

$\Rightarrow \forall k:\ p_k = \frac{\lambda}{2}$ (konst.) $\Rightarrow \forall k:\ p_k = \frac{1}{365}$

48 a $P(1 \text{ korrekt}) = \frac{1}{2}\cdot\frac{1}{3} + \frac{1}{2}\cdot\frac{2}{3} = \frac{1}{2}$ $P(5 \text{ korrekte}) = \left(\frac{1}{2}\right)^5$

b $P(1 \text{ korrekt}) = \frac{1}{3}\cdot\frac{1}{3} + \frac{2}{3}\cdot\frac{2}{3} = \frac{5}{9}$ $P(5 \text{ korrekte}) = \left(\frac{5}{9}\right)^5$

c $P(1 \text{ korrekt}) = 0\cdot\frac{1}{3} + 1\cdot\frac{2}{3} = \frac{2}{3}$ $P(5 \text{ korrekte}) = \left(\frac{2}{3}\right)^5$

50 a $P(A) = \frac{3}{6} = \frac{1}{2}$ $P(B) = \frac{4}{6} = \frac{2}{3}$ $P(C) = \frac{3}{36} = \frac{1}{12}$

b $P(A\cap B\cap C) = \frac{1}{36} = P(A)P(B)P(C)$

$P(A\cap B) = \frac{12}{36} = \frac{1}{3} = P(A)P(B)$

$P(A\cap C) = \frac{3}{36} = \frac{1}{12} \neq P(A)P(C)$

$P(B\cap C) = \frac{1}{36} \neq P(B)P(C)$

A, B og C er afh.

1.5

55 $\quad - - - \;\to\; \left\{ \begin{array}{l} - - - \\ - - \circ \end{array} \right. \;\to\; \left\{ \begin{array}{l} - - - \\ \circ\; - \\ - - \circ \end{array} \right. \;\to\; \circ$

a $\quad P = \frac{4}{5} \cdot \frac{2}{3} \cdot 1 = \frac{8}{15}$

b $\quad P = \frac{2n-2}{2n-1} \cdot \frac{2n-4}{2n-3} \cdot \ldots \cdot \frac{2}{3} \cdot 1$

1.6

65 $\quad P = \sum_{k=1}^{6} \frac{1}{6} \left(\frac{1}{2}\right)^k = \frac{1}{6} \cdot \frac{1}{2} \, \frac{1-\left(\frac{1}{2}\right)^6}{1-\frac{1}{2}} = \frac{1}{6}\left(1 - \frac{1}{64}\right) = \frac{21}{128} \approx 0{,}1641$

67 $\quad \boxed{\text{1 sort}} \quad \boxed{\begin{array}{l}\text{9 sorte}\\ \text{10 hv.}\end{array}} \qquad P = \frac{1}{2} \cdot 1 + \frac{1}{2}\,\frac{9}{19} = \frac{14}{19} \approx 0{,}7366$ (maks.)

(min.: $\frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$)

68 $\quad$ ARKANSAS $\qquad$ 3 A'er $\quad$ 2 S'er $\quad$ 3 andre (forsk.)

3 bogstaver fjernes og sættes tilfældigt tilbage

$$P(\text{korrekt}) = P(3 \text{ ens}) \cdot 1 + P(2 \text{ ens}) \frac{1}{3} + P(3 \text{ forsk.}) \frac{1}{6}$$

$$= \frac{1}{\binom{8}{3}} \cdot 1 + \frac{3 \cdot 5 + 1 \cdot 6}{\binom{8}{3}} \cdot \frac{1}{3} + \frac{\binom{8}{3} - (1 + 3 \cdot 5 + 1 \cdot 6)}{\binom{8}{3}} \cdot \frac{1}{6}$$

$$= \frac{1}{56}\left(1 + \frac{21}{3} + \frac{34}{6}\right) = \frac{41}{168} \approx 0{,}2440$$

71 $\quad P(X=k) = \dfrac{\binom{4}{k}\binom{48}{4-k}}{\binom{52}{4}} \,, \qquad P(Y=n \mid X=k) = \dfrac{\binom{4}{n}\binom{48}{k-n}}{\binom{52}{k}}$

$$P(X=k,\, Y=n) = \frac{\binom{4}{n}\binom{48}{k-n}\binom{4}{k}\binom{48}{4-k}}{\binom{52}{k}\binom{52}{4}} \,, \qquad k = 0, \ldots, 4$$

$$n = 0, \ldots, k$$

1.6

73 $P(A|B)$ er et sands. mål, jf. sætn. 1.5.1 s. 32

Total sands. ved betingning med $C|B$ og $C^c|B$:

$$P(A|B) = P((A|B)|(C|B))\,P(C|B) + P((A|B)|(C^c|B))\,P(C^c|B)$$
$$= P(A|(B\cap C))\,P(C|B) + P(A|B\cap C^c)\,P(C^c|B)$$

77 a Sikrer, at B og G er uafh.

b $$P(B|G) = \frac{1\cdot\frac{k}{m}}{1\,\frac{k}{m} + 1\left(1-\frac{k}{m}\right)} = \frac{k}{m}$$

$$1 - P(B|G) = 1 - \frac{k}{m} = \frac{m-k}{m}$$

81 $P(\text{modt.} \mid \text{sendt}) = 0{,}9$ $P(0) = \frac{2}{3}$ $P(1) = \frac{1}{3}$ 0,1 uafh.

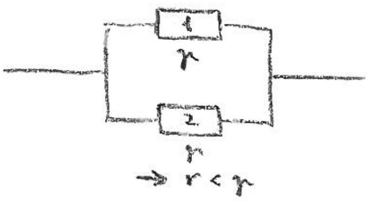
a $P(1\text{-}0 \text{ modt} \mid 1\text{-}0 \text{ sendt}) = 0{,}9 \cdot 0{,}9 = 0{,}81$

b $$P(1 \text{ sendt} \mid 1 \text{ modt.}) = \frac{0{,}9\cdot\frac{1}{3}}{0{,}9\cdot\frac{1}{3} + 0{,}1\cdot\frac{2}{3}} = \frac{9}{11} \approx 0{,}8182$$

$$P(0 \text{ sendt} \mid 0 \text{ modt}) = \frac{0{,}9\cdot\frac{2}{3}}{0{,}9\cdot\frac{2}{3} + 0{,}1\cdot\frac{1}{3}} = \frac{18}{19} \approx 0{,}9474$$

$$P(1\text{-}0 \text{ sendt} \mid 1\text{-}0 \text{ modt.}) = \frac{9}{11}\cdot\frac{18}{19} = \frac{162}{209} \approx 0{,}7751$$

88



a $P(2) = p^2 + (1-p)r$ (total sands.)

b $P(\text{syst.}) = 1 - (1-p)(1-r)$

c $$P(1|2^c) = \frac{P(1\cap 2^c)}{P(2^c)} = \frac{p(1-p)}{1-(p^2+(1-p)r)} = \frac{p}{1+p-r}$$

97 $q = 1(1-p) + q^2 p$, jf. eks 1.6.17 s. 62 nederst

$$\Rightarrow q^2 - \frac{1}{p}q + \frac{1-p}{p} = 0 \Rightarrow q = \begin{cases} \frac{1-p}{p} \\ (1) \text{ fremmedrod} \end{cases}$$