2.6

62 $T \sim e\left(\frac{1}{10}\right)$

a $\operatorname{Var} T = \frac{1}{\left(\frac{1}{10}\right)^2} = 100$

b $P(T \le 5) = F(5) = 1 - e^{-\frac{1}{10}\cdot 5} = 1 - e^{-\frac{1}{2}} = 0,3955$

c $P(T \le 30 \mid T > 25) = P(T \le 5) = 1 - e^{-\frac{1}{2}} = 0,3955$

d $P(T > ET) = P\left(T > \frac{1}{\frac{1}{10}}\right) = P(T > 10) = e^{-\frac{1}{10}10} = e^{-1}$

$= 0,3679$

65 $X \sim e(\lambda)$, $m = 58 \Rightarrow \frac{\ln 2}{\lambda} = 58 \Rightarrow \lambda = \frac{\ln 2}{58} = 0,0120$

a $P(X > 30) = e^{-\frac{\ln 2}{58}30} = 0,6987$

b $P(X \le 60 \mid X > 30) = P(X \le 30) = 1 - P(X > 30) = 0,3013$

c $EX = \frac{1}{\lambda} = \frac{58}{\ln 2} = 83,6763$

$\operatorname{Var} X = \frac{1}{\lambda^2} = \left(\frac{58}{\ln 2}\right)^2 = 7001,7253$

66 $T \sim e(\lambda)$, $X = [T] + 1$

i $P(X = k) = P(k-1 \le T < k) = 1 - e^{-k\lambda} - \left(1 - e^{-(k-1)\lambda}\right)$

$= e^{-(k-1)\lambda}(1 - e^{-\lambda}) = (e^{-\lambda})^{k-1}(1 - e^{-\lambda})$, $k = 1, 2, \dots$

$X \sim g(1 - e^{-\lambda})$

ii Hvis tiden deles op i ækvidistante intervaller, så angiver X det tidsinterval, hvor komponenten fejler.

68 X, Y ekspeditionstider for kunder under betjening, $X, Y \sim e(\lambda)$ uafh.

$P(X > Y) = P(Y > X)$ pga. sym. $\Rightarrow P(X > Y) = \frac{1}{2}$

Z ekspeditionstid for ny kunde, $Z \sim e(\lambda)$, Z, X uafh.

$P(X > y + Z \mid X > y) = P(X > Z) \Rightarrow P(X > Y + Z \mid X > Y) = P(X > Z)$

$P(\text{den går sidst}) = P(X > Y + Z \mid X > Y)\, P(X > Y) = P(X > Z)\, P(X > Y)$

$= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

2.7

70 $T = t + 0{,}1 + X \quad , \quad X \sim N(0; 0{,}01)$

a $\quad T \sim N(t + 0{,}1 ; 0{,}01)$

b $\quad t = 11{,}5 \quad , \quad P(T \le 11{,}5) = \Phi\left(\frac{11{,}5 - (11{,}5 + 0{,}1)}{0{,}1}\right) = \Phi(-1) = 0{,}1586$

c $\quad P(t - 0{,}05 < T < t + 0{,}05) = \Phi\left(\frac{t + 0{,}05 - (t + 0{,}1)}{0{,}1}\right) - \Phi\left(\frac{t - 0{,}05 - (t + 0{,}1)}{0{,}1}\right)$

$\quad = \Phi(-0{,}5) - \Phi(-1{,}5) = 0{,}30854 - 0{,}06681 = 0{,}2417$

73 $\ell = 100 \text{ cm} \quad , \quad X \sim N(\mu, 2)$

a $\quad Y = \begin{cases} X \mid X < 100 \\ X - 100 \mid X \ge 100 \end{cases}$

b $\quad EY = \mu \, P(X < 100) + (\mu - 100) \, P(X \ge 100)$

$\quad = \mu \, \Phi\left(\frac{100-\mu}{\sqrt{2}}\right) + (\mu - 100)\left(1 - \Phi\left(\frac{100-\mu}{\sqrt{2}}\right)\right)$

$\quad = \mu - 100\left(1 - \Phi\left(\frac{100-\mu}{\sqrt{2}}\right)\right)$

$\frac{d(EY)}{d\mu} = 1 - 100\left(-\frac{1}{\sqrt{2\pi}\sqrt{2}} e^{-\frac{(100-\mu)^2}{2 \cdot 2}}\right)\left(-\frac{1}{\sqrt{2}}\right) = 1 - \frac{50}{\sqrt{2\pi}} e^{-\frac{(100-\mu)^2}{4}}$

$\quad = 0$ for $e^{-\frac{(100-\mu)^2}{4}} = \frac{\sqrt{2\pi}}{50} \Rightarrow \mu = 103{,}46$ cm

74 $X \quad , \quad F(x) = 1 - e^{-x^2}, \; x \ge 0 \quad , \quad f(x) = 2x e^{-x^2}, \; x \ge 0$

$EX = \int_0^\infty x \cdot 2x e^{-x^2} dx = \int_0^\infty 2x^2 e^{-x^2} dx \quad , \quad x = \frac{u}{\sqrt{2}} \; , \; x^2 = \frac{u^2}{2} \; , \; dx = \frac{1}{\sqrt{2}} du$

$\quad = \int_0^\infty u^2 e^{-\frac{u^2}{2}} \frac{1}{\sqrt{2}} du = \sqrt{\pi} \int_0^\infty u^2 \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du$

$\quad = \frac{\sqrt{\pi}}{2} \int_{-\infty}^\infty u^2 \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du = \frac{\sqrt{\pi}}{2} E[U^2] \quad , \quad U \sim N(0,1)$

$\quad = \frac{\sqrt{\pi}}{2}$, idet $E[U^2] = \text{Var}\,U + (EU)^2 = 1 + 0^2 = 1$

75 i $\quad X \sim N(\mu, \sigma^2)$

$P(|X - \mu| > k\sigma) = P\left(\left|\frac{X-\mu}{\sigma}\right| > \frac{k\sigma}{\sigma}\right) = P\left(\left|\frac{X-\mu}{\sigma}\right| > k\right)$

$\quad = \Phi(-k) + (1 - \Phi(k)) = 2\,\Phi(-k)$

ii $\quad$ Chebychevs ulighed : $P(|X - \mu| > k\sigma) \le \frac{1}{k^2}$

k	$2\Phi(-k)$	$\frac{1}{k^2}$
1	0,3173	1,0000
2	0,0455	0,2500
3	0,0027	0,1111
4	0,0001	0,0625
5	0,0000	0,0400

2.8

81 $X \sim \Gamma(\alpha, \lambda)$, $Y = \lambda X$, $y = \lambda x \Leftrightarrow x = \frac{1}{\lambda} y$, $\frac{dx}{dy} = \frac{1}{\lambda}$

$$f_Y(y) = \frac{\lambda^\alpha}{\Gamma(\alpha)} \left(\frac{1}{\lambda} y\right)^{\alpha-1} e^{-\lambda(\frac{1}{\lambda} y)} \left|\frac{1}{\lambda}\right| = \frac{1}{\Gamma(\alpha)} y^{\alpha-1} e^{-y}$$

$Y \sim \Gamma(\alpha, 1)$

82 $X \sim$ Cauchy , $f(x) = \frac{1}{\pi(1+x^2)}$

a $F(x) = \int_{-\infty}^{x} \frac{1}{\pi(1+t^2)}\, dt = \frac{1}{\pi}\left[\operatorname{arctg} t\right]_{-\infty}^{x} = \frac{1}{\pi} \operatorname{arctg} x + \frac{1}{2}$

b $Y = \frac{1}{X}$, $y = \frac{1}{x} \Leftrightarrow x = \frac{1}{y}$, $\frac{dx}{dy} = -\frac{1}{y^2}$

$$f_Y(y) = \frac{1}{\pi\left(1 + \frac{1}{y^2}\right)} \left|-\frac{1}{y^2}\right| = \frac{1}{\pi(1+y^2)}$$ dvs $Y \sim$ Cauchy

84 a $P(X \le x) = P(X \le x \mid \text{type I})\, P(\text{type I})$

$+ P(X \le x \mid \text{type II})\, P(\text{type II})$

$= (1 - e^{-x})\frac{1}{2} + (1 - e^{-2x})\frac{1}{2}$

$= 1 - \frac{1}{2}(e^{-x} + e^{-2x})$

b $P(\text{type I} \mid X > t) = \dfrac{P(X > t \mid \text{type I})\, P(\text{type I})}{P(X > t)}$

$= \dfrac{\frac{1}{2} e^{-t}}{\frac{1}{2}(e^{-t} + e^{-2t})} = \dfrac{1}{1 + e^{-t}}$

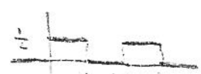
2.9

85 a $f(x) = \frac{1}{x^2}$, $x \ge 1$

$\mu = \int_1^\infty x \frac{1}{x^2} dx = \int_1^\infty \frac{1}{x} dx = [\ln x]_0^\infty = \infty$

$F(x) = \int_1^x \frac{1}{t^2} dt = \left[-\frac{1}{t}\right]_1^x = 1 - \frac{1}{x}$, $1 - \frac{1}{m} = \frac{1}{2} \Rightarrow m = 2$

$x_m = 1$

b $f(x) = \frac{1}{2}$,  , $\mu = 1{,}5$, $m \in [1;2]$

$x_m \in [0,1] \cup [2;3]$

c $f(x) = 2(1-x)$, $0 < x < 1$

$\mu = \int_0^1 x\, 2(1-x)\, dx = \left[x^2 - \frac{2x^3}{3}\right]_0^1 = 1 - \frac{2}{3} = \frac{1}{3}$

$F(x) = \int_0^x 2(1-t)\, dt = [2t - t^2]_0^x = 2x - x^2$, $2m - m^2 = \frac{1}{2} \Rightarrow m = 1 - \frac{\sqrt{2}}{2}$

$x_m = 0$

d $f(x) = \frac{1}{\pi(1+x^2)}$, μ ikke def. , $m = 0$, $x_m = 0$

2.10

93 X, $X = k$, $k = 0, 1, \ldots$, $r(k) = \dfrac{P(X=k)}{P(X \geq k)}$

a $r(k) = \dfrac{P(X=k)}{P(X \geq k)} = \dfrac{P(X=k, X \geq k)}{P(X \geq k)} = P(X = k \mid X \geq k)$

b $P(X=k) = \frac{1}{6}$, $k = 1, \ldots, 6$

$r(k) = \dfrac{\frac{1}{6}}{1 - \frac{k-1}{6}} = \dfrac{1}{7-k}$, $k = 1, \ldots, 6$

$r(6) = 1$ ~ $k = 6$ er højeste levealder

95 a $f(t) = 2t$, $0 \leq t \leq 1$, $F(t) = t^2$, $0 \leq t \leq 1$

$r(t) = \dfrac{2t}{1-t^2}$, $0 \leq t \leq 1$

b $f(t) = \dfrac{1}{t^2}$, $1 \leq t < \infty$, $F(t) = 1 - \dfrac{1}{t}$, $1 \leq t < \infty$

$r(t) = \dfrac{\frac{1}{t^2}}{1 - (1 - \frac{1}{t})} = \dfrac{1}{t}$, $1 \leq t < \infty$

c $f(t) = 2t \exp(-t^2)$, $0 \leq t < \infty$, $F(t) = 1 - \exp(-t^2)$, $0 \leq t < \infty$

$r(t) = \dfrac{2t \exp(-t^2)}{1 - (1 - \exp(-t^2))} = 2t$, $0 \leq t < \infty$

97 L: levetid af insekt, $m = 2$, $r(t) = at^2$, $0 \leq t < \infty$

a $G(t) = \exp\left(-\int_0^t a u^2 \, du\right) = \exp\left(-\frac{1}{3} a t^3\right)$

$\exp\left(-\frac{1}{3} a \, 2^3\right) = \frac{1}{2} \Rightarrow a = \dfrac{3 \ln 2}{8} = 0{,}2599$

b $P(L > 3) = G(3) = \exp\left(-\frac{1}{3} \frac{3 \ln 2}{8} 3^3\right) = \exp\left(-\frac{27 \ln 2}{8}\right)$

$= 0{,}0964$