Vink til opgavesæt 20

7.4   40   a   Mellemresultat:

$$\gamma_{i,i+1} = i+\alpha \quad , \quad \gamma_{i,i-1} = 3i \quad , \quad i = 0,1,\ldots$$

b   Bemærk, at $\dfrac{\gamma_{10,11}}{\gamma_{10,9}+\gamma_{10,11}} = \dfrac{\gamma_{10,11}}{\gamma_{10,9}+\gamma_{10,11}}$.

c   Husk, at $E_0[\tau_0] = \dfrac{1}{\pi_0}$.

41   Rækkeoperationer

$$G^T = \begin{bmatrix} -1 & 3 & 0 & 0 & \cdots \\ 1 & -4 & 6 & 0 & \cdots \\ 0 & 1 & -8 & 9 & \cdots \\ 0 & 0 & 2 & -12 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \end{bmatrix} = \begin{bmatrix} -1 & 3 & 0 & 0 & \cdots \\ 0 & -1 & 6 & 0 & \cdots \\ 0 & 0 & -2 & 9 & \cdots \\ 0 & 0 & 0 & -3 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \end{bmatrix}$$

43   a, b   Benyt eks. 7.4.13 side 458.

44   a   M/M/1/1 , $\rho = 1$

b   M/M/1/1 , $\rho = \frac{1}{2}$

c   M/M/1/2 , $\rho = 1$

d   M/M/2/2 , $\rho = \frac{1}{2}$

45   a   Rækkeoperationer

$$G^T = \begin{bmatrix} -1 & 2 & 0 & 0 \\ 0 & -3 & 2 & 0 \\ 1 & 0 & -2 & 2 \\ 0 & 1 & 0 & -2 \end{bmatrix} \sim \cdots \sim \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

b   Rækkeoperationer

$$G^T = \begin{bmatrix} -1 & 2 & 0 & 0 \\ 0 & -3 & 2 & 0 \\ 1 & 0 & -\frac{3}{2} & 2 \\ 0 & 1 & \frac{1}{2} & -2 \end{bmatrix} \sim \cdots \sim \begin{bmatrix} 7 & 0 & 0 & -16 \\ 0 & 7 & 0 & -8 \\ 0 & 0 & 7 & -12 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

46 a –

b Rækkeoperationer

$$G^T = \begin{bmatrix} -1 & 1 & 0 & 0 \\ \frac{1}{2} & -2 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{3}{2} & 1 \\ 0 & \frac{1}{2} & \frac{1}{2} & -1 \end{bmatrix} \sim \cdots \sim \begin{bmatrix} 5 & 0 & 0 & -4 \\ 0 & 5 & 0 & -4 \\ 0 & 0 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

47 a Mellemresultat : $\gamma_{12} = 2,1$

Rækkeoperationer

$$G^T = \begin{bmatrix} -3 & 0,5 & 0 \\ 3 & -2,6 & 0,5 \\ 0 & 2,1 & -0,5 \end{bmatrix} \sim \cdots \sim \begin{bmatrix} 6 & -1 & 0 \\ 0 & 21 & -5 \\ 0 & 0 & 0 \end{bmatrix}$$

b –

48 Mellemresultat : $\sum_{n=1}^{\infty} \frac{\lambda_0 \cdots \lambda_{n-1}}{\mu_1 \cdots \mu_n} = e^{\frac{\lambda}{\mu}} - 1$

49 –

51 Bemærk, at $T_n = \sum_{j=0}^{n} V_j$, $V_j \sim e(\mu)$, $j = 0, \ldots, n$, uafh.

$$\Rightarrow \quad T_n \sim \Gamma(n+1, \mu)$$

Udregn $P(T > t) = \sum_{n=0}^{\infty} P(T_n > t)\, P(X_t = n)$ ved

udnyttelse af $P(T_n > t) = e^{-\mu t} \sum_{k=0}^{n} \frac{(\mu t)^k}{k!}$, jf.

formel side 137 nederst

Mellemresultat : $P(T > t) = e^{-(1-\rho)\mu t}$