Ensidig variansanalyse – igen!

Kaldes også enfaktor variansanalyse

$y_{ij} = \beta_i + u_{ij}$, $i = 1, \dots, m$ $\left\{ \begin{array}{l} \text{faktorens} \\ \text{niveauer} \end{array} \right.$

$j = 1, \dots, n$

Sæt

$$y = (y_{11}, \dots, y_{1n}, \dots, y_{i1}, \dots, y_{in}, \dots, y_{m1}, \dots, y_{mn})$$

(dvs $y \in \mathbb{R}^{mn}$)

og

$$X = \begin{bmatrix} 1_n & 0 & \dots & 0 \\ 0 & 1_n & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1_n \end{bmatrix} \quad mn \times m$$

Model:

$$y = X\beta + u, \quad U \sim N_{mn}(0, \sigma^2 I_{mn})$$

$$\beta = (\beta_1, \beta_2, \dots, \beta_m) \in \mathbb{R}^m$$

Udregninger

$$X^T X = \begin{bmatrix} 1_n^T & 0^T & \dots & 0^T \\ 0^T & 1_n^T & \dots & 0^T \\ \vdots & \vdots & & \vdots \\ 0^T & 0^T & \dots & 1_n^T \end{bmatrix} \begin{bmatrix} 1_n & 0 & \dots & 0 \\ 0 & 1_n & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & 1_n \end{bmatrix}$$

$$= \begin{bmatrix} n & 0 & \dots & 0 \\ 0 & n & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & n \end{bmatrix} = n I_m$$

$$\hat{\beta} = (X^T X)^{-1} X^T y = \frac{1}{n} I_m \begin{bmatrix} 1_n^T & 0^T & \cdots & 0^T \\ 0^T & 1_n^T & \cdots & 0^T \\ \vdots & & & \\ 0^T & 0^T & \cdots & 1_n^T \end{bmatrix} y$$

$$= \frac{1}{n} \begin{bmatrix} n\bar{y}_{1\cdot} \\ n\bar{y}_{2\cdot} \\ \vdots \\ n\bar{y}_{m\cdot} \end{bmatrix} = \begin{bmatrix} \bar{y}_{1\cdot} \\ \bar{y}_{2\cdot} \\ \vdots \\ \bar{y}_{m\cdot} \end{bmatrix}$$

Hypotese

$$H_0: \beta_1 = \beta_2 = \cdots = \beta_m \quad *$$

kan repræsenteres ved $H\beta = 0$, hvor

$$H = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 & 0 \\ 1 & 0 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & 0 & 0 & & -1 & 0 \\ 1 & 0 & 0 & & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1_{m-1} & -I_{m-1} \end{bmatrix}$$

$(m-1) \times m$

(dvs. $\beta_1 - \beta_2 = 0 \wedge \beta_1 - \beta_3 = 0 \wedge \cdots \wedge \beta_1 - \beta_m = 0$)

Udregninger

$$H(X^T X)^{-1} H^T = H\left(\frac{1}{n} I_m\right) H^T = \frac{1}{n} H H^T$$

$$= \frac{1}{n} \begin{bmatrix} 1_{m-1} & -I_{m-1} \end{bmatrix} \begin{bmatrix} 1_{m-1}^T \\ -I_{m-1} \end{bmatrix}$$

$$= \frac{1}{n} \left[1_{m-1} 1_{m-1}^T + I_{m-1} \right]$$

$$= \frac{1}{n} \left[I_{m-1} + 1_{m-1} 1_{m-1}^T \right]$$

* H_1: mindst et β_i afvigende

$$K = \left(H (X^T X)^{-1} H^T \right)^{-1}$$

$$= n \left(I_{m-1}^{-1} - \frac{1}{1 + 1_{m-1}^T I_{m-1}^{-1} 1_{m-1}} I_{m-1}^{-1} 1_{m-1} 1_{m-1}^T I_{m-1}^{-1} \right)$$

$$= n \left(I_{m-1} - \frac{1}{\underbrace{1+m-1}_{m}} 1_{m-1} 1_{m-1}^T \right)$$

$$\| \hat{\mu} - \hat{\mu}_0 \|^2 = y^T P_H y = (H\hat{\beta})^T K H\hat{\beta}$$

$$= n \left(\| H\hat{\beta} \|^2 - \frac{1}{m} \left(1_{m-1}^T H\hat{\beta} \right)^2 \right)$$

$$H\hat{\beta} = \begin{bmatrix} \bar{y}_{1\cdot} - \bar{y}_{2\cdot} \\ \bar{y}_{1\cdot} - \bar{y}_{3\cdot} \\ \vdots \\ \bar{y}_{1\cdot} - \bar{y}_{m\cdot} \end{bmatrix}$$

$$1_{m-1}^T H\hat{\beta} = (m-1)\bar{y}_{1\cdot} - \sum_{i=2}^{m} \bar{y}_{i\cdot}$$

$$= m\bar{y}_{1\cdot} - \sum_{i=1}^{m} \bar{y}_{i\cdot} = m(\bar{y}_{1\cdot} - \bar{y}_{\cdot\cdot})$$

$$\| \hat{\mu} - \hat{\mu}_0 \|^2 = n \left(\sum_{i=2}^{m} (\bar{y}_{1\cdot} - \bar{y}_{i\cdot})^2 - \frac{1}{m} \left(m(\bar{y}_{1\cdot} - \bar{y}_{\cdot\cdot}) \right)^2 \right)$$

$$= n \left(\sum_{i=1}^{m} (\bar{y}_{1\cdot} - \bar{y}_{i\cdot})^2 - m(\bar{y}_{1\cdot} - \bar{y}_{\cdot\cdot})^2 \right)$$

$$= n \Big(m\bar{y}_{1\cdot}^2 - 2m\bar{y}_{1\cdot}\bar{y}_{\cdot\cdot} + \sum_{i=1}^{m} \bar{y}_{i\cdot}^2$$

$$- m\bar{y}_{1\cdot}^2 + 2m\bar{y}_{1\cdot}\bar{y}_{\cdot\cdot} - m\bar{y}_{\cdot\cdot}^2 \Big)$$

$$= n \sum_{i=1}^{m} (\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})^2$$

(lettere : $\hat{\mu}_i = \bar{y}_{i\cdot} \wedge \hat{\mu}_{i0} = \bar{y}_{\cdot\cdot}$

$$\Rightarrow \|\hat{\mu} - \hat{\mu}_0\|^2 = \sum_{i=1}^{m} \sum_{j=1}^{n} (\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})^2$$

$$= n \sum_{i=1}^{m} (\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})^2 \quad)$$

Alternativ formulering af hypotesen

$$\tilde{H}\mu = 0 ,$$

$$\tilde{H} = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -1 & \cdots & 0 & 0 \\ \vdots \\ 0 & 0 & 0 & \cdots & 1 & -1 \end{bmatrix} \quad (m-1) \times m$$

$$= [\, I_{m-1} \;\; 0 \,] - [\, 0 \;\; I_{m-1} \,]$$

$$\tilde{H} (X^T X)^{-1} \tilde{H}^T = \tilde{H} \tfrac{1}{n} I_m \tilde{H}^T = \tfrac{1}{n} \tilde{H} \tilde{H}^T$$

$$= \tfrac{1}{n} \left([\, I_{m-1} \;\; 0 \,] - [\, 0 \;\; I_{m-1} \,] \right) \left(\begin{bmatrix} I_{m-1} \\ 0^T \end{bmatrix} - \begin{bmatrix} 0^T \\ I_{m-1} \end{bmatrix} \right)$$

$$= \tfrac{1}{n} \left(I_{m-1} + 0 - \begin{bmatrix} 0^T & 0 \\ I_{m-2} & 0 \end{bmatrix} - \begin{bmatrix} 0 & I_{m-2} \\ 0 & 0^T \end{bmatrix} + 0 + I_{m-1} \right)$$

$$= \tfrac{1}{n} \begin{bmatrix} 2 & -1 & 0 & \cdots & 0 & 0 \\ -1 & 2 & -1 & \cdots & 0 & 0 \\ 0 & -1 & 2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 2 & -1 \\ 0 & 0 & 0 & \cdots & -1 & 2 \end{bmatrix} = A \quad (m-1) \times (m-1)$$

$$a_{ij} = \begin{cases} \frac{2}{n} & \text{for } i = j \\ -\frac{1}{n} & \text{for } |i-j| = 1 \end{cases}$$

$$K = \left(\tilde{H} (X^T X)^{-1} \tilde{H}^T \right)^{-1} = A^{-1}$$

$$k_{ij} = \begin{cases} \frac{n}{m}\, i\,(m-j) & \text{for } i \le j \\ \frac{n}{m}\,(m-i)\,j & \text{for } i \ge j \end{cases}$$

kontrol: $\sum_{l=1}^{m-1} a_{il} k_{lj} = \delta_{ij}$, idet

$i < j$: $\frac{1}{m}\left(-(i-1)(m-j) + 2i(m-j) - (i+1)(m-j)\right) = 0$

$i = j$: $\frac{1}{m}\left(-(i-1)(m-(i-1)) + 2i(m-i) - i(m-(i-1))\right) = 1$

$i > j$: $\frac{1}{m}\left(-(m-(i-1))j + 2(m-i)j - (m-(i+1))j\right) = 0$

$$\tilde{H}^T K \tilde{H} = \left(\begin{bmatrix} I_{m-1} \\ 0^T \end{bmatrix} - \begin{bmatrix} 0^T \\ I_{m-1} \end{bmatrix} \right) K \left([I_{m-1}\ 0] - [0\ I_{m-1}] \right)$$

$$= \left(\begin{bmatrix} K \\ 0^T \end{bmatrix} - \begin{bmatrix} 0^T \\ K \end{bmatrix} \right) \left([I_{m-1}\ 0] - [0\ I_{m-1}] \right)$$

$$= \begin{bmatrix} K & 0 \\ 0^T & 0 \end{bmatrix} - \begin{bmatrix} 0 & K \\ 0 & 0^T \end{bmatrix} - \begin{bmatrix} 0^T & 0 \\ K & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0^T \\ 0 & K \end{bmatrix}$$

udregning af ij-te element

$i < j$: $\frac{n}{m}\big(i(m-j) - (i-1)(m-j)$
$\qquad - i(m-(j-1)) + (i-1)(m-(j-1))\big) = -\frac{n}{m}$

$i = j$: $\frac{n}{m}\big(i(m-i) - (m-i)(i-1)$
$\qquad - (i-1)(m-i) + (m-(i-1))(i-1)\big) = \frac{n}{m}(m-1)$

$i > j$: $\frac{n}{m}\big((m-i)j - (m-(i-1))j$
$\qquad - (m-i)(j-1) + (m-(i-1))(j-1)\big) = -\frac{n}{m}$

$$\tilde{H}^T K \tilde{H} = \frac{n}{m}\left(m I_m - 1_m 1_m^T \right)$$

$$= n \left(I_m - \frac{1}{m} 1_m 1_m^T \right)$$

$$\hat{\beta}^T \tilde{H}^T K \tilde{H} \hat{\beta} = n \left(\sum_{i=1}^{m} \bar{y}_{i\cdot}^2 - \frac{1}{m} \left(\sum_{i=1}^{m} \bar{y}_{i\cdot} \right)^2 \right)$$

$$= n \left(\sum_{i=1}^{m} \bar{y}_{i\cdot}^2 - m \bar{y}_{\cdot\cdot}^2 \right)$$

$$= n \sum_{i=1}^{m} \left(\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot} \right)^2$$

dvs. $\| \hat{\mu} - \hat{\mu}_0 \|^2 = n \sum_{i=1}^{m} (\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})^2$

altså samme resultat som på side 3

Residualkvadratsummen

$$\| y - \hat{\mu} \|^2 = \| y - X\hat{\beta} \|^2$$

$$= \sum_i \sum_j (y_{ij} - \bar{y}_{i\cdot})^2$$

Test af H_0 ($H\beta = 0$ eller $\tilde{H}\beta = 0$)

$$f_{obs} = \frac{\frac{1}{m-1} \| \hat{\mu} - \hat{\mu}_0 \|^2}{\frac{1}{m(n-1)} \| y - \hat{\mu} \|^2}$$

$$= \frac{\frac{n}{m-1} \sum (\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})^2}{\frac{1}{m(n-1)} \sum_{i=1}^{m} \sum_{j=1}^{n} (y_{ij} - \bar{y}_{\cdot\cdot})^2} \text{, hvor}$$

$F \sim F(m-1, m(n-1))$ under H_0

Alternativ parametrisering

$$y_{ij} = \bar{\mu} + \alpha_i + u_{ij} \quad , \quad \bar{\mu} = \frac{1}{n} \sum \beta_i$$

$$\alpha_i = \beta_i - \bar{\mu} \quad \Rightarrow \quad \sum_i \alpha_i = \sum_i \beta_i - n\bar{\mu}$$
$$= n\bar{\mu} - n\bar{\mu} = 0$$

α_i'erne kaldes faktorens virkning på de forskellige niveauer

Da $\sum_i \alpha_i = 0$, har vi

$$\alpha_m = -\alpha_1 - \alpha_2 - \dots - \alpha_{m-1} \quad (\ast)$$

hvorefter

$$y = Z\gamma + u \quad , \quad \gamma = (\bar{\mu}, \alpha_1, \dots, \alpha_{m-1}) \in R^m$$

$$Z = \begin{bmatrix} 1_n & 1_n & 0 & \dots & 0 & 0 \\ 1_n & 0 & 1_n & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1_n & 0 & 0 & \dots & 1_n & 0 \\ 1_n & 0 & 0 & \dots & 0 & 1_n \\ 1_n & -1_n & -1_n & \dots & -1_n & -1_n \end{bmatrix} \quad mn \times m$$

$$C(Z) = C(X) \quad \Rightarrow \quad Z(Z^T Z)^{-1} Z^T = X(X^T X)^{-1} X^T$$
$$\Rightarrow \quad Z\hat{\gamma} = X\hat{\beta} \text{ for alle } y$$

Test i den reparametriserede model:

$$H_* \gamma = 0 \quad , \quad H_* = [\,0 \quad I_{m-1}\,] \quad (m-1) \times m$$

$$(\text{dvs. } \alpha_1 = \alpha_2 = \dots = \alpha_m = 0)$$

Variansanalysetabel

Variation	kvadrat-summer	frihedsgrader
mellem niveauer	$n \sum_i (\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})^2$	$m-1$
residual	$\sum_i \sum_j (y_{ij} - \bar{y}_{i\cdot})^2$	$m(n-1)$
total	$\sum_i \sum_j (y_{ij} - \bar{y}_{\cdot\cdot})^2$	$mn-1$

Blokforsøg

(tosidig variansanalyse uden gentagelser)

Eks. Høstudbytte

Faktor : Kunstgødning

Blokke : Bonitet af jorden

Model : $y_{ij} = \bar{\mu} + \alpha_i + \gamma_j + u_{ij}$

$$\sum_i \alpha_i = \sum_j \gamma_j = 0$$

NB! Modellen forudsætter additivitet af faktorvirkning og blokvirkning, dvs. en evt. vekselvirkning kan ikke afdækkes.

Tabel

variation	kvadratsummer	frihedsgrader
mellem niveauer	$n \sum_i (\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})^2$	$m-1$
mellem blokke	$m \sum_j (\bar{y}_{\cdot j} - \bar{y}_{\cdot\cdot})^2$	$n-1$
residual	$\sum_i \sum_j (y_{ij} - \bar{y}_{i\cdot} - \bar{y}_{\cdot j} + \bar{y}_{\cdot\cdot})^2$	$(m-1)(n-1)$
total	$\sum_i \sum_j (y_{ij} - \bar{y}_{\cdot\cdot})^2$	$mn-1$

Blandet model

Sammenligning af to niveauer justeret for værdier af en kvantitativ forklarende variabel

Model:

$$y_i = \bar{\mu} + \alpha + \gamma x_i + u_i \quad , \quad i = 1, \ldots, n$$

$$y_i = \bar{\mu} - \alpha + \gamma x_i + u_i \quad , \quad i = n+1, \ldots, 2n$$

$$y = X\beta \quad , \quad \beta = (\bar{\mu}, \alpha, \gamma)$$

$$X = \begin{bmatrix} 1 & 1 & x_1 \\ \vdots & \vdots & \vdots \\ 1 & 1 & x_n \\ 1 & -1 & x_{n+1} \\ \vdots & \vdots & \vdots \\ 1 & -1 & x_{2n} \end{bmatrix} \qquad 2n \times 3$$

Beregninger

$$X^T X = \begin{bmatrix} 2n & 0 & \Sigma x_i \\ 0 & 2n & \Sigma x_{iA} - \Sigma x_{iB} \\ \Sigma x_i & \Sigma x_{iA} - \Sigma x_{iB} & \Sigma x_i^2 \end{bmatrix}$$

$(x_{iA} := x_i , \ i = 1, \ldots, n \ ; \ x_{iB} := x_i , \ i = n+1, \ldots, 2n)$

$$X^T y = \begin{bmatrix} \Sigma y_i \\ \Sigma y_{iA} - \Sigma y_{iB} \\ \Sigma x_i y_i \end{bmatrix}$$

Løsning af normalligningerne

$\hat{\mu}$ udtrykt ved $\hat{\gamma}$:

$$2n\bar{\mu} + (\Sigma x_i)\gamma = \Sigma y_i \Rightarrow \hat{\mu} = \bar{y} - \bar{x}\hat{\gamma}$$

$\hat{\alpha}$ udtrykt ved $\hat{\gamma}$:

$$2n\alpha + (\Sigma x_{iA} - \Sigma x_{iB})\gamma = \Sigma y_{iA} - \Sigma y_{iB}$$

$$\Rightarrow \hat{\alpha} = \tfrac{1}{2}(\bar{y}_A - \bar{y}_B - (\bar{x}_A - \bar{x}_B)\hat{\gamma})$$

Løsning for $\hat{\gamma}$:

$$(\Sigma x_i)\bar{\mu} + (\Sigma x_{iA} - \Sigma x_{iB})\alpha + (\Sigma x_i^2)\gamma = \Sigma x_i y_i$$

$$\Rightarrow \Sigma x_i(\bar{y} - \bar{x}\gamma) + (\Sigma x_{iA} - \Sigma x_{iB})\tfrac{1}{2}(\bar{y}_A - \bar{y}_B - (\bar{x}_A - \bar{x}_B)\gamma) - (\Sigma x_i^2)\gamma = \Sigma x_i y_i$$

$$\Rightarrow \left(-\Sigma x_i\bar{x} - \tfrac{1}{2}(\Sigma x_{iA} - \Sigma x_{iB})(\bar{x}_A - \bar{x}_B) + \Sigma x_i^2\right)\gamma = \Sigma x_i y_i - \Sigma x_i \bar{y} - \tfrac{1}{2}(\Sigma x_{iA} - \Sigma x_{iB})(\bar{y}_A - \bar{y}_B)$$

$$\Rightarrow \left(-\tfrac{n}{2}(\bar{x}_A + \bar{x}_B)^2 - \tfrac{n}{2}(\bar{x}_A - \bar{x}_B)^2 + \Sigma x_{iA}^2 + \Sigma x_{iB}^2\right)\gamma = \Sigma x_{iA}y_{iA} + \Sigma x_{iB}y_{iB} - \tfrac{n}{2}(\bar{x}_A + \bar{x}_B)(\bar{y}_A + \bar{y}_B) - \tfrac{n}{2}(\bar{x}_A - \bar{x}_B)(\bar{y}_A - \bar{y}_B)$$

$$\Rightarrow \left(\Sigma x_{iA}^2 - n\bar{x}_A^2 + \Sigma x_{iB}^2 - n\bar{x}_B^2\right)\gamma = \Sigma(x_{iA} - \bar{x}_A)y_{iA} + \Sigma x_{iB} - \bar{x}_B)y_{iB}$$

$$\Rightarrow \hat{\gamma} = \frac{\Sigma(x_{iA} - \bar{x}_A)y_{iA} + \Sigma(x_{iB} - \bar{x}_B)y_{iB}}{\Sigma(x_{iA} - \bar{x}_A)^2 + \Sigma(x_{iB} - \bar{x}_B)^2}$$

Test af $H_0: \alpha = 0$, $H_1: \alpha \neq 0$

H_0 repræsenteres af $H\beta = 0$, $H = [0\ 1\ 0]$, $q = 1$

$H\hat{\beta} = \hat{\alpha}$

$$K = (H(X^TX)^{-1}H^T)^{-1} = \left(((X^TX)^{-1})_{2,2}\right)^{-1}$$

$$= \left(\frac{1}{\det X^TX}\left((-1)^{2+2}\det((X^TX)_{22})\right)^T\right)^{-1} = \frac{\det X^TX}{\det((X^TX)_{22})}$$

$$\det X^T X = 2n \begin{vmatrix} 1 & 0 & \bar{x} \\ 0 & 2n & \Sigma x_{iA} - \Sigma x_{iB} \\ \Sigma x_i & \Sigma x_{iA} - \Sigma x_{iB} & \Sigma x_i^2 \end{vmatrix}$$

$$= 2n\,(2n \Sigma x_i^2 - (\Sigma x_{iA} - \Sigma x_{iB})^2 + \bar{x}(-2n \Sigma x_i))$$

$$= 4n^2 (\Sigma x_{iA}^2 + \Sigma x_{iB}^2 - \tfrac{n}{2}(\bar{x}_A - \bar{x}_B)^2 - \tfrac{n}{2}(\bar{x}_A + \bar{x}_B)^2)$$

$$= 4n^2 (\Sigma x_{iA}^2 - n\bar{x}_A^2 + \Sigma x_{iB}^2 - n\bar{x}_B^2)$$

$$\det (X^T X)_{22} = 2n(\Sigma x_i^2 - \Sigma x_i \bar{x}) = 2n(\Sigma x_i^2 - 2n\bar{x}^2)$$

$$f_{obs} = \frac{\hat{\alpha}\, 4n^2 (\Sigma x_{iA}^2 - n\bar{x}_A^2 + \Sigma x_{iB}^2 - n\bar{x}_B^2)\, \hat{\alpha}}{2n(\Sigma x_i^2 - 2n\bar{x}^2)\, s^2}$$

$$= \frac{\hat{\alpha}^2\, 2n\, (\Sigma (x_{iA} - \bar{x}_A)^2 + \Sigma (x_{iB} - \bar{x}_B)^2)}{s^2 \Sigma (x_i - \bar{x})^2}$$

$$s^2 = \frac{1}{2n-3} \Sigma (y_i - \hat{y})^2$$

$F \sim F(1, 2n-3)$ under H_0

(alt. benyttes $T \sim t(2n-3)$, $T^2 = F$)

Evt. model med ikke-parallelle regressions-linier, dvs. med et γ_A forskellig fra et γ_B. I så fald må gælde

$E[Y_{iA} - Y_{iB}] = 2\alpha - (\gamma_A - \gamma_B)$, ikke konstant, dvs. en model med vekselvirkning, resultater vanskelige at fortolke.

Modelselektion

Kræver ideelt set et dybtgående kendskab til det fænomen, der skal belyses.

Langtfra altid tilfældet! Derfor et samspil mellem den baggrundsviden, der foreligger, og dataanalyse.

Hvis data ikke passer på en lineær model, kan der overvejes

- modeludvidelse
  - tilføj fx x^2 eller $\ln x$
  - Box-Cox transformation, dvs.

    $T(y,\lambda) = X\beta + u$, hvor

    $$T(y,\lambda) = \begin{cases} \dfrac{y^\lambda - 1}{\lambda} & \text{for } \lambda \neq 0 \\ \ln y & \text{for } \lambda = 0 \end{cases}$$

- grafiske metoder (bl.a. residualplot)

- ikke-parametriske metoder

Specielt problem

- 'outliers'

  brug evt. robuste metoder