Foldning

Betragt $f, g : \mathbb{R} \to \mathbb{R}$, hvor f og g er stykkevis kontinuerte, og hvor mindst én af funktionerne er absolut integrabel, den anden begrænset.

Funktionen $f * g$, hvor

$$f * g\,(t) = \int_{-\infty}^{\infty} f(u)\, g(t-u)\, du,$$

kaldes foldningen af f og g.

Hvis f og/eller g er identisk 0 i visse intervaller, bliver intervallet, der skal integreres over, en delmængde af $]-\infty; \infty[$.

Eksempel 1.

$$f(x) = g(x) = \begin{cases} 0 & \text{for } x < 0 \\ e^{-x} & \text{for } x \geqq 0 \end{cases}$$

$$f * g\,(t) = \int_0^t e^{-u} e^{-(t-u)} du = \int_0^t e^{-t} du = te^{-t}, \quad 0 \leqq t < \infty \qquad \square$$

Eksempel 2.

$$f(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ e^{-x} & \text{for} \quad x \geqq 0 \end{cases} \qquad g(x) = \begin{cases} e^{x} & \text{for} \quad x \leqq 0 \\ 0 & \text{for} \quad x > 0 \end{cases}$$

$$f * g\,(t) = \int_t^{\infty} e^{-u} e^{t-u} du = e^t \int_t^{\infty} e^{-2u} du = e^t \left[-\tfrac{1}{2} e^{-2u}\right]_t^{\infty} = \tfrac{1}{2} e^{-t},$$

$$0 \leqq t < \infty \qquad \square$$

Eksempel 3.

$$f(x) = g(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \quad -\infty < x < \infty$$

$$\begin{aligned} f * g\,(t) &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} \frac{1}{\sqrt{2\pi}} e^{-\frac{(t-u)^2}{2}} du = \frac{1}{2\pi} e^{-\frac{t^2}{2}} \int_{-\infty}^{\infty} e^{-(u^2 - tu)} du \\ &= \frac{1}{2\pi} e^{-\frac{t^2}{4}} \int_{-\infty}^{\infty} e^{-\left(u-\frac{t}{2}\right)^2} du\,; \qquad v = \sqrt{2}\left(u - \frac{t}{2}\right),\ dv = \sqrt{2} du \\ &= \frac{1}{2\pi} e^{-\frac{t^2}{4}} \int_{-\infty}^{\infty} e^{-\frac{v^2}{2}} \frac{1}{\sqrt{2}} dv = \frac{1}{2\sqrt{\pi}} e^{-\frac{t^2}{4}} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-v^2}{2}} dv \\ &= \frac{1}{2\sqrt{\pi}} e^{-\frac{t^2}{4}}, \quad -\infty < t < \infty \qquad \square \end{aligned}$$

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