Obstruction theory for time-reversal symmetric topological insulators and $\mathbb{Z}_2$ invariants

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Abstract: Time-reversal symmetric (TRS) topological insulators are gapped periodic quantum systems enjoying a fermionic TRS, that is, the time-reversal operator squares to $-\mathbf{1}$. Recently, these crystalline materials have attracted the attention of the condensed matter and mathematical physics communities, both for their peculiar transport properties and for their topological quantum phases, which are distinguished by $\mathbb{Z}_2$-valued indices.

We provide an interpretation of these indices as topological invariants, characterizing the obstruction to the existence of smooth, periodic and TRS Bloch frames (or equivalently of a basis of localized TRS Wannier functions) for the spectral eigenprojectors of the Hamiltonian of the system, in 2 and 3 dimensions and allowing for an arbitrary (even) number of energy bands. No model-dependent assumption is needed for this characterization, only the symmetries (periodicity, TRS) of the quantum system.

In dimension 2, the $\mathbb{Z}_2$ invariant can also be linked to the problem of finding a logarithm for families of unitary matrices which enjoy a suitable TRS constraint.

The talk is based on joint works with Domenico Fiorenza and Gianluca Panati, and with Horia Cornean and Stefan Teufel.