# Remarks on $L^p$-boundedness of wave operators for Schrödinger operators with threshold singularities

K. Yajima*

**Abstract**

The continuity property in Lebesgue spaces $L^p(\mathbb{R}^m)$ of wave operators $W_\pm$ for Schrödinger operator $H = -\Delta + V$ on $\mathbb{R}^m$ is considered when $H$ is of exceptional type, i.e. $\mathcal{N} = \{u \in \langle x \rangle^{-s} L^2(\mathbb{R}^m)\colon (1 + (-\Delta)^{-1}V)u = 0\} \neq \{0\}$. It has recently been proved by Goldbereg and Green for $m \geq 5$ that $W_\pm$ are bounded in $L^p(\mathbb{R}^m)$ for $1 \leq p < m/2$, the same holds for $1 \leq p < m$ if all $\phi \in \mathcal{N}$ satisfy $\int_{\mathbb{R}^m} V\phi dx = 0$ and, for $1 \leq p < \infty$ if in addition $\int_{\mathbb{R}^m} x_i V\phi dx = 0$, $i = 1, \ldots, m$. We make the results for $p > m/2$ more precise and prove in particular that these conditions are also necessary for the stated properties of $W_\pm$. We also prove that, for $m = 3$, $W_\pm$ are bounded in $L^p(\mathbb{R}^3)$ for $1 < p < 3$ and that the same holds for $1 < p < \infty$ if and only if all $\phi \in \mathcal{N}$ satisfy $\int_{\mathbb{R}^3} V\phi dx = 0$ and $\int_{\mathbb{R}^3} x_i V\phi dx = 0$, $i = 1, \ldots, 3$, simultaneously.

*Department of Mathematics, Gakushuin University, 1-5-1 Mejiro, Toshima-ku, Tokyo 171-8588, Japan. Supported by JSPS grant in aid for scientific research No. 22340029