Schedule

Type 2

8:15 – 10:00 Lectures in lecture room G5-109

10:00 – 11:30 Exercises in the group rooms (on your own)

11:30 – 12:00 Questions and answers.

Lecture

Repetition and Perspectives

A short survey on curves and their invariants.

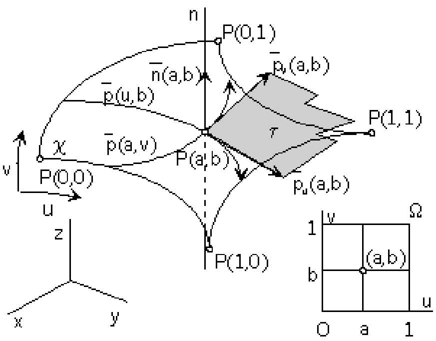
Introduction to surfaces

Intutitively a surface is a subset $S \subset \mathbb{R}^3$ which *looks like the plane, locally*. To make sense of this intuition, one needs to be careful both with the term *local* and *looks like*.
Charts: Almost as with a physical atlas of the surface of our earth, we start by defining individual charts, which formalize the local comparison of S to the plane. Such a chart (or surface patch or parametrization) is a map σ from a subset of the plane to S which

- defines a *homeomorphism* (continuous inverse!) between an open set in the (2D-) plane and a (relatively) open subset of the surface S;
- is a *smooth* map of two variables u, v into 3D-space $\mathbb{R}^3$
- has a differential of *rank* 2, i.e., of maximal rank. [1]

We look at several examples; among them surfaces that are given as the graph of a function of two variables.

Atlas: We need enough charts so that they cover the entire surface; a collection of patches is then called an *atlas* for the smooth – or regular – surface S.



Obr. 4.1

Implicitly given surfaces - as level sets

As with curves, it is often easier to describe a surface as the set S of solutions of an equation $f(x, y, z) = 0$ in 3 variables x, y, z – a level set for f. Is such a level set S a regular surface? Not always, you have already found a counter-example examining the equation $xyz = 0$.

The condition that the gradient $\nabla f(x, y, z) \neq$ *at any* $(x, y, z) \in S$ is sufficient to ensure that S is indeed a smooth surface. The proof proceeds by applying the *implicit function theorem*. In the textbook, the implicit function theorem is not assumed; this is why a longer proof, using solely the inverse function theorem is inserted.

[1] There are several equivalent formulations for this requirement (check your linear algebra books). It ensures (together with the homeomorphism requirement) that the surface has tangent *planes*.

FAJSTRUP@MATH.AAU.DK — 9940 8848 — FREDRIK BAJERSVEJ 7G
9220 AALBORG ØST

Fiddling with parametrizations

Composing a regular chart with a local plane diffeomorphism provides us with an alternative regular chart – a *reparametrization*, comparable with a change of coordinates in Linear Algebra.

The main subtlety in the definition of a surface patch σ is that it *does not make sense* to require that the inverse map σ^{-1} is *smooth*.[2] This difficulty can be overcome using several tools to be explained mainly in the next lecture. Remark that the inverse function theorem from analysis is our work horse throughout.

References

AP Ch. 4.1 – 4.2, 5.1

FR Ch. 2.1, pp. 53 – 61.

Wikipedia Smooth surface

Applets

- Parametrized surfaces
- Geometry: Gallery of Surfaces
- IMAGINARY

Exercises

AP 4.2.3, 4.2.1, 4.1.2, 4.2.2[3], 4.2.5

Next activity

Time Thursday, September 22, 8:15 – 12:00.

Type 1

Content Transition functions. Graph coordinate systems. Curves on surfaces. Smooth maps on surfaces.

[2] Why? Because the inverse is not defined on an open subset of a Euclidean space. And this is where differentiability has been defined in your analysis courses.

[3] Postpone questions about the transition functions until the next lecture!

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