Schedule

Type: 4

8:15 – 12:00: *No lecture.* Presentations by group members after preparation. If questions arise, please send me an email. Then I will take them up the next time we meet.

Presentation on smooth maps

You are asked to prepare a short presentation (at most 15 minutes) on *Smooth maps between surfaces and their derivatives* – or differentials. The talk should include

- definitions of both concepts;
- an explanation why the concepts are *well-defined*; and
- a description of the differential of the smooth map by a matrix (with respect to given surface patches for both manifolds); and
- in particular why this settles that the differential is a linear map.

Try to make your presentation as self-contained as possible. Illustrative diagrams may help to clarify the main ideas.

You should choose one or two group members who will talk in front of the others, and at least one whose task it is to critize – in a constructive way – the presentation with respect to content, (possible) mistakes/misunderstandigs and explanatory "performance". I would be pleased to attend some of the presentations and to give feedback, as well.

As your main source, you may choose one of the references mentioned on the lecture plan for Activity 6 and 7 or also your lecture notes – or a combination.

Exercises

Leftovers from previous activities

Torus Consider the torus T with the parametrization given in Exercise 4.2.5 (p. 80). For given parameters (θ, φ), let $\mathbf{p} = \sigma(\theta, \varphi)$.
Determine a basis for the tangent space $T_{\mathbf{p}}S$ and the unit normal vector $\mathbf{N}_\sigma(\mathbf{p})$.
Is the torus an orientable surface?

A torus map Given a matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ with *integer* coefficients $a, b, c, d \in \mathbf{Z}$. A map $F_A : T \to T$ can be defined by $F_A(\sigma(\theta, \varphi)) = \sigma((\theta, \varphi)A)$ – multiplication of the row vector (θ, φ) by the matrix A resulting in a new row vector.
Check that F_A is a well-defined (θ and φ are only well-defined up to a factor of 2π) smooth map.
Determine the curves $F_A(\gamma_i)$, $1 \le i \le 2$, with γ_i given by the "equator" $\theta = 0$, resp. the "meridian" $\varphi = 0$.
Determine the derivative (or differential) $D_{\mathbf{p}}F_A$ – at an arbitrary point $\mathbf{p} \in T$ as a matrix with respect to the same parametrization σ on domain and codomain T.

FAJSTRUP@MATH.AAU.DK — 9940 8855 — FREDRIK BAJERSVEJ 7G, 9220 AALBORG ØST

Next Activity

Date Tuesday, October 4, 8:15 – 12:00.

Type 2

Content The first fundamental form.
(AP), ch. 6.1 – 6.2.

FAJSTRUP@MATH.AAU.DK 9940 8855 FREDRIK BAJERSVEJ 7G
9220 AALBORG ØST