Schedule

Type 3

8:15 – 9:30 Short repetition and lecture

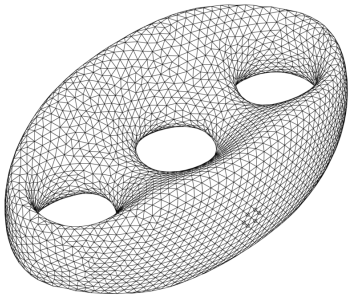
9:30 – 12:00 Exercise session in group rooms; some help from the lecturer available.

Repetition. Perspectives

Compact surfaces, triangulations.
Euler characteristic as a topological invariant.
Gauss-Bonnet theorem for curvilinear polygons and for compact surfaces.

Lecture

Which compact surfaces can the Gauss-Bonnet theorem be applied to? Orientable compact surfaces can be classified up to diffeomorphism by their *genus*, loosely speaking the number of "holes" in such a surface. To make that definition rigorous is not that easy and beyond the scope of the course. But it is easy to construct a surface T_g of a given genus g:
T_0 is the 2-sphere S^2, T_1 is the torus, and T_g, $g \geq 1$, is obtained by joining g tori/torusses together.



A triangulation of T_3

How about the Euler characteristics? By exhibiting a particular triangulation, one determines that $\chi(T_g) = 2 - 2g$; and by the Gauss-Bonnet theorem, the total curvature of T_g is equal to $4\pi(1-g)$ – positive for a spherelike surface, 0 for a toruslike surface, and negative for all others.

The lecture will end with a few indications concerning

- hyperbolic geometry
- minimal surfaces (with mean curvature $H = 0$)
- differential geometry in higher dimensions

References

AP A. Pressley, *Elementary Differential Geometry*, ch. 13.2 –4.

Wikipedia Gauss-Bonnet theorem

Wikipedia Euler characteristic

Wikipedia Genus

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Exercises

Leftovers from previous exercise sessions

Geodesic 2-gons Let S be an orientable surface of negative or zero Gaussian curvature. Show that two geodesics γ_1, γ_2 which start at some point $p \in S$ cannot meet again at a point $q \in S$ in such a say that the traces of γ_1 and γ_2 together form a 2-gon that constitutes the boundary of a simple region of S.

Geodesics on an elliptic surface Show: Two simple closed geodesics γ_1, γ_2 on a compact surface of positive Gaussian curvature must intersect.[1]

Next activity

Question Session Time and place to be announced. (Monday Jan. 16 is suggested)

Oral Exam Tuesdag January 17 Here is a document specifying the curriculum, focus areas for presentations and a description of the schedule for each exam.

[1] Use that the surface is homeomorphic to a sphere. What do two non-intersecting closed curves on a sphere bound, together?