Location

Department of Mathematical Sciences, Fredrik Bajersvej 7G, G5-109.

Main Reference:

[HSD]: M.W. Hirsch, S. Smale, R.L. Devaney, *Dynamical Systems, Differential Equations & an Introduction to Chaos*, 2nd ed., Elsevier, 2004.

Supplementary reading

- J.H. Hubbard, B.H. West, *Differential Equations: A Dynamical Systems Approach. Higher-Dimensional Systems*, Texts in Applied Mathematics **18**, Springer-Verlag, 1995.
- from the internet: M. Grinfeld, Higher-dimensional Dynamical Systems

Please bring a laptop that can connect to your department's internet server via AAU-net.

Overall structure

We plan to organise the sessions to include both lectures and hands-on exercise sessions with a daily scheme like

9	– 10	Lecture 1
10	– 11	Exercise session 1
11	– 12	Lecture 2
12	– 12:30	Lunch break
12:30	– 13:20	Lecture 3
13:20	– 14:10	Exercise session 2
14:10	– 15	Lecture 4

Please take the exercise sessions seriously. The examples treated and the reasoning asked of you will make the lectures more concrete and they will test your understanding of the material.

We will start with chapter 3 of the book, hence we expect you to work your way through the first two chapters. You should not dwell too long on section 1.5 on the Poincaré map; if you do, make sure to correct the differentiation of f with

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respect to x_0 to differentiation with respect to φ as you may have seen from the list of corrections on the homepage for the book.

Introduction. Simple 2D linear dynamical systems

Thu., 8.9., 9 – 12

Startup

Lectures

Aims and Content

In HSD chapter 2, explicit solutions of a linear planar system $X' = AX$ were constructed using eigenvalues and eigenvectors of A and these solutions were combined to give all solutions to such a system; at least in the case when the eignevalues were real. In fact, a similar analysis is possible for complex eigenvalues. The qualitative behaviour of the solutions depends on the eigenvalues - are they real or complex, what are their signs or are they zero? To get an idea of the qualitative behaviour, we will study systems with three types of matrices,

$$A = \begin{pmatrix} \lambda & 0 \\ 0 & \mu \end{pmatrix}, A = \begin{pmatrix} \alpha & \beta \\ -\beta & \alpha \end{pmatrix}$$

$$A = \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}$$

in detail.

References:

[HSD] 3.1, 3.2 and 3.3.

Exercises:

[HSD] p.37 Ex. 2, 3, 7 and 12 (Hint: This can be proven in a direct way studying the characteristic polynomial, but for the afficionados, you might appreciate that this is a consequence of the Cayley Hamilton theorem, which says that a matrix is a root of its characteristic polynomial.)

[HSD] p. 57 Ex. 1 using the solver referred to on the course webpage.

Classification and behaviour of 2D linear dynamical systems.

Thu., 8.9., 12:30 – 15

Lectures

Aims and Content

Dynamical sytems associated to general invertible (2×2)-matrices show the same performance (look-alike phase planes) as those explained in the previous standard examples. They correspond to

- sinks, sources or saddles in the case of two independent eigenvectors associated to real eigenvalues;
- spiral sinks or sources in the case of two (conjugate) complex eigenvalues
- and a transition case in case of an eigenvalue of algebraic multiplicity *two* with a *one*-dimensional associated eigenspace.

It turns out, that the *trace* and the *determinant* of a (2×2)-matrix in common determine which type of dynamical system the matrix describes.

References:

[HSD], ch. 3.4 & 4.1

Exercises:

[HSD], 3.1, 3.2(i)(iii)(v), 3.4.

Dynamical Classification.

Fri.,9.9.,9-12

Lectures

Aims and Content

We are often interested in comparison of global (long term) behavior of solutions of different differential equations. We say that two systems of differential equations are equivalent if their solutions share the same fate. For example a linear system with a spiral sink and a linear system with (real) sink share the same fate: They tend to the origin as time goes to infinity. We say that these systems are conjugate. A precise definition of this notion will be introduced during the lecture. We shall give examples and a criterion for dynamical systems to be conjugate.

References:

[HSD] 4.2

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Exercises:

Ex. 4 on page 72, Ex. 5a on page 72,
Ex. 6 on page 73.

Examples of higher dimensional systems.

Fri.,9.9. 12.30-15

Lectures

Aims and Content

For higher dimensional linear systems, eigenvalues and eigenvectors still carry the essential information of the system.The solution methods from 3.4 will work again. We study some particular 3 dimensional examples to seehow solutions behave in higher dimensions.

References:

[HSD] 6.1

Exercises:

[HSD] p.135, ex. 1 and 2.

Next Block

Thursday and Friday 22-23 September

Before we meet again, we expect you to read through chapter 5, which is probably well known for most of you. Moreover, as part of the evaluation of the course, you are required to work through the exploration 4.3 in groups of 2 or 3 people. We expect you to hand in a short report on your work. Deadline for this is Monday Sep.19.

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