Location

Department of Mathematical Sciences, Fredrik Bajersvej 7G, G5-109.
October 6.

Main Reference:

[HSD]: M.W. Hirsch, S. Smale, R.L. Devaney, *Dynamical Systems, Differential Equations & an Introduction to Chaos*, 2nd ed., Elsevier, 2004.

Supplementary reading

- J.H. Hubbard, B.H. West, *Differential Equations: A Dynamical Systems Approach. Higher-Dimensional Systems*, Texts in Applied Mathematics **18**, Springer-Verlag, 1995.
- from the internet: M. Grinfeld, Higher-dimensional Dynamical Systems

Non-linear saddles, sinks and sources

Lecture

Thursday, October 6, morning session

Aims and Content

In a planar nonlinear system, $X' = F(X)$, the local flow around an equilibrium point X_0, $F(X_0) = 0$, is *conjugate* to the flow of the linearized system $X' = DF_{X_0}(X)$, if the linearized system is *hyperbolic*. This result generalizes to higher dimensional systems in a staightforward way in the cases where the real parts of all eigenvalues have the same sign, i.e., for sinks and sources. The higher dimensional analogue to a saddle point occurs when k eigenvalues have negative real parts and $n-k$ have positive real parts. The stable and unstable curves from dimension 2 will be replaced by stable and unstable *submanifolds* of dimension k and $n-k$. The linearized system has an unstable k-dimensional subspace and a stable $n-k$-dimensional subspace. Again, the non-linear system is conjugate to the linerarized system, at least in a neighborhood of the equilibrium point.

References:

[HSD], ch. 8.2 – 8.3, pp. 165 – 174.

Exercises:

[HSD], ch, 8, 8.5. Please use one of the plot tools to illustrate some particular cases - to find the the null clines, equilibrium point(s), their linearization etc.

Stability and Bifurcations.

Lecture

Thursday, October 6, afternoon session

Aims and Content

Stability can be given at least two different meanings for a dynamical system: An equilibrium point may be stable, asymptotically stable – or unstable.

A different stability issue is the stability of the system as such under change of the parameters defining it. Smooth variation of the setup of an experiment will correspond to a variation of parameters giving rise to a family of systems $X' = F_a(X)$, where a represents the varying parameter. A bifurcation occurs at a, if the behaviour of the system is significantly different for values $a - \varepsilon$ and values $a + \varepsilon$. For first order differential equations, such abrupt changes can only occur at (equilibrium) points X_0 where $DF_a(X_0) = 0$.

References:

[HSD], ch. 8.4 and 8.5.

Exercises:

We did not have time to go into a closer study of the behaviour of a non-linear system between the equilibrium points. This is the theme of this exercise.

1. The system $x' = x^2 - 1$ and $y' = -xy$ has two saddle points (where?) Prove that there is a solution curve which is a stable curve for one saddle point and an unstable curve for the other. Hint: It runs on the x-axis.

2. The system $x' = -2x(x-1)(2x-1)$ and $y' = -2y$ has three equilibrium points. Classify the linearization at these three points and use a plot tool to see how the solutions behave.

3. Same question for the system $x' = y$ and $y' = -x^3 + x$. Here, the unstable curves of the saddle point turn around and come back as the stable curve - they are *homoclinic orbits*.

In Chapter 9, the last two examples are studied on p.205 and p.209 as examples of a gradient system and a Hamiltonian system.

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If you have time, then exercise 9.1 gives many examples of behaviour in between equilibrium points. And 9.7 provides examples of gradient flow systems.

Course evaluation

During the afternoon session, we would like to evaluate the course. Please think about positive aspects that you would like us to keep doing for the next time, we give the course – and also, which changes you would suggest.

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