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**Solutions to exercize Kreyszig 6.2.11**

$$y'' + ay' + by = r(t) \quad ; \quad y(0) = 1\,;\, y'(0) = 31{}^1\!/\!_2$$
$$a = 3\,;\, b = {}^9\!/\!_4$$
$$r(t) = 9t^3 + 64 \rightarrow \text{by table} \rightarrow R(s) = {}^{54}\!/\!_{s^4} + {}^{64}\!/\!_s$$

General form of the solution is:

$$(s^2 + as + b)Y(s) = (s + a)y(0) + y'(0) + R(s)$$

So in this case:

$$(s^2 + 3s + {}^9\!/\!_4)Y(s) = (s+3)1 + 31{}^1\!/\!_2 + R(s)$$
$$(s + {}^3\!/\!_2)^2 Y(s) = (s + {}^3\!/\!_2) + \frac{33s^4 + 64s^3 + 54}{s^4}$$
$$Y(s) = \frac{1}{(s + {}^3\!/\!_2)} + \frac{33s^4 + 64s^3 + 54}{(s + {}^3\!/\!_2)^2 s^4}$$

By partial fraction decomposition and solving, this becomes (see $*$):

$$Y(s) = \frac{1}{(s + {}^3\!/\!_2)} + \frac{A}{s^4} + \frac{B}{s^3} + \frac{C}{s^2} + \frac{D}{s} + \frac{E}{(s + {}^3\!/\!_2)^2} + \frac{F}{(s + {}^3\!/\!_2)}$$
$$\text{with } A = 24; B = -32; C = 32; D = 0; E = 1; F = 0$$

So:

$$Y(s) = \frac{1}{(s + {}^3\!/\!_2)} + \frac{24}{s^4} + \frac{-32}{s^3} + \frac{32}{s^2} + \frac{1}{(s + {}^3\!/\!_2)^2}$$

By the table and the first shifting theorem it follows that:

$$y(t) = e^{-3/2t} + 4t^3 - 16t^2 + 32t + te^{-3/2t}$$

$*$Partial fraction decomposition by:

$$\frac{A}{s^4} + \frac{B}{s^3} + \frac{C}{s^2} + \frac{D}{s} + \frac{E}{(s + {}^3\!/\!_2)^2} + \frac{F}{(s + {}^3\!/\!_2)} = \frac{33s^4 + 64s^3 + 54}{(s + {}^3\!/\!_2)^2 s^4}$$
$$A(s^2 + 3s + {}^9\!/\!_4) + B(s^3 + 3s^2 + {}^9\!/\!_4 s) + C(s^4 + 3s^3 + {}^9\!/\!_4 s^2) +$$
$$+ D(s^5 + 3s^4 + {}^9\!/\!_4 s^3) + E(s^4) + F(s^5 + {}^9\!/\!_4 s^4) = 33s^4 + 64s^3 + 54$$

Ordering terms by power of s:

$$s^0 : {}^9\!/\!_4 = 54 \Rightarrow \qquad A = 24$$

$$s^1 : 3A + {}^9\!/\!_4 B = 0 \Rightarrow \qquad B = -32$$

$$s^2 : A + 3B + {}^9\!/\!_4 C = 0 \Rightarrow \qquad C = 32$$

$$s^3 : B + 3C + {}^9\!/\!_4 D = 64 \Rightarrow \qquad D = 0$$

$$s^4 : C + 3D + E + {}^3\!/\!_2 F = 33 \Rightarrow \qquad E = 1$$

$$s^5 : D + F = 0 \Rightarrow \qquad F = 0 \rightarrow \text{(put in } s^4 \text{ equation)}$$