



## Exercises: Basics of probability theory

### Exercise 1

A fair coin is tossed  $n$  times (where  $n$  is a given positive integer).

1. Specify the state space  $\Omega$  and the probability measure  $P$  for all possible realisations.
2. Let  $A$  be the event “the coin shows up with both a head and a tail” and let  $B$  be the event that “there is at most one tail”. Determine  $P(A)$ ,  $P(B)$ , and  $P(A \cap B)$ .
3. Are  $A$  and  $B$  independent events?

### Exercise 2

A number  $X$  is picked uniformly at random on the interval  $[0, 1]$ , that is for any  $I \subseteq [0, 1]$ ,  $P(X \in I) = \text{length of } I$ . We say that  $X$  is *uniformly distributed between 0 and 1* and write  $X \sim \text{unif}(0, 1)$ .

1. Specify the distribution function, density function, mean, and variance of  $X$ .
2. What is the probability that the first decimal of  $X$  is equal to 1.

### Exercise 3

A random variable  $X$  is said to follow an *exponential distribution with parameter*  $\lambda > 0$  if  $X$  has density

$$f_X(x) = \lambda \exp(-\lambda x), \quad x > 0$$

(meaning that  $f_X(x) = 0$  if  $x \leq 0$ ).

1. Determine the distribution function and the mean of  $X$ .
2. For any numbers  $s > 0$  and  $t > 0$ , find  $P(X > t + s | X > s)$  and interpret the result.