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Importance sampling for unnormalized densities

Let the situation be as in Section 5 in "A short diversion into the theory of Markov chains, with a view to Markov chain Monte Carlo methods". However, assume that the target density π is only known up to proportionality, i.e. $\pi(x) = \pi_u(x)/c_\pi$ where π_u is a known unnormalized density and c_π is an unknown normalizing constant. Moreover, assume that the instrumental density is $g(x) = g_u(x)/c_g$ where g_u is a known unnormalized density and c_π is a (possibly unknown) normalizing constant. Then the estimator in Section 5,

$$\tilde{\theta} = \frac{1}{n+1} \sum_{i=m}^{m+n} h(Y_i) \frac{\pi(Y_i)}{g(Y_i)} = \frac{1}{n+1} \sum_{i=m}^{m+n} h(Y_i) \frac{\pi_u(Y_i)}{g_u(Y_i)} \Big/ \frac{c_\pi}{c_g} \tag{1}$$

depends on the ratio c_π / c_g of unknown normalizing constants. Note that

$$\frac{c_\pi}{c_g} = \frac{\int \pi_u(x)\,\mathrm{d}x}{c_g} = \int \frac{\pi_u(x)}{g_u(x)} \frac{g_u(x)}{c_g}\,\mathrm{d}x = \int \frac{\pi_u(x)}{g_u(x)} g(x)\,\mathrm{d}x = E\left[\frac{\pi_u(Y)}{g_u(Y)}\right]$$

where Y follows the instrumental density g. Thus it is possible to estimate c_π/c_g, using the sample $Y_1, \ldots, Y_n$ (either an iid sample or a sample from an irreducible Markov chain with invariant density g):

$$\frac{c_\pi}{c_g} \approx \frac{1}{n+1} \sum_{i=m}^{m+n} \frac{\pi_u(Y_i)}{g_u(Y_i)}. \tag{2}$$

Finally, combining (1) and (2), we obtain the following consistent estimator of θ:

$$\bar{\theta} = \frac{\frac{1}{n+1} \sum_{i=m}^{m+n} h(Y_i) \pi_u(Y_i)/g_u(Y_i)}{\frac{1}{n+1} \sum_{i=m}^{m+n} \pi_u(Y_i)/g_u(Y_i)}.$$