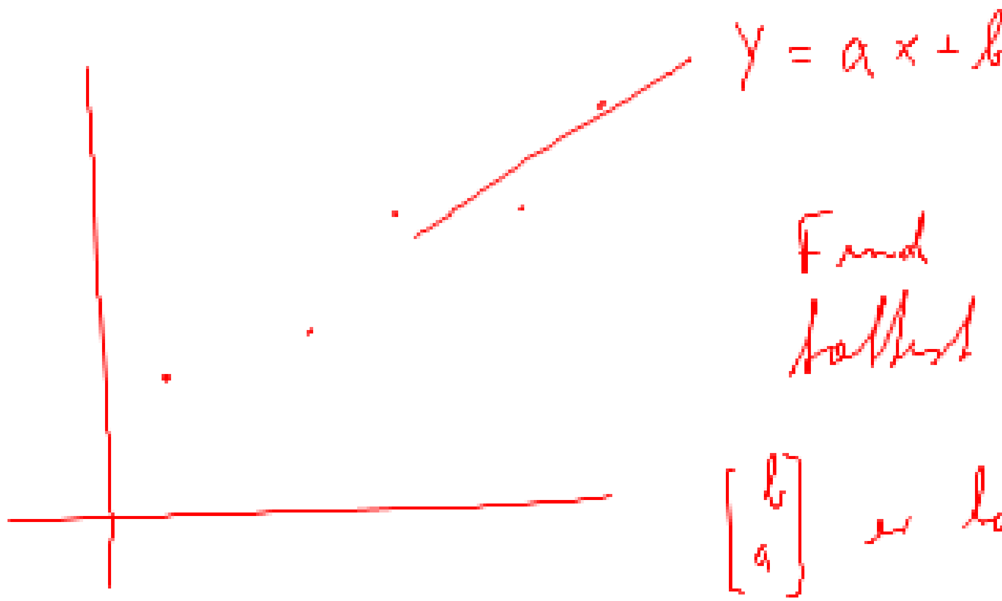


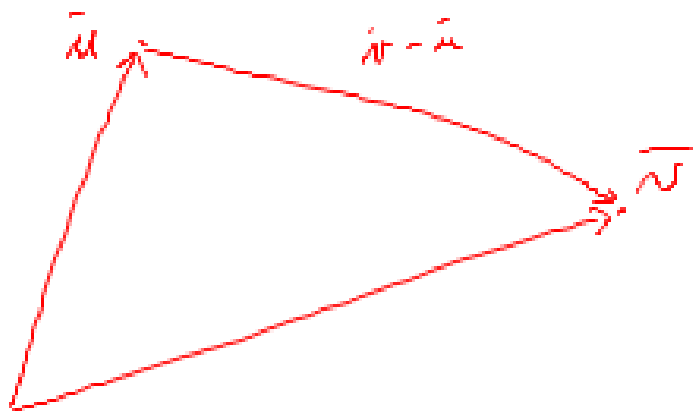
Find linie $y = ax + b$

tættest på $(x_1, y_1), \dots, (x_n, y_n)$

$\begin{bmatrix} b \\ a \end{bmatrix}$ er løsning til

$$\begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

eller 'næsten løsning'



$$\operatorname{dist}(\bar{u}, \bar{v}) = \|\bar{v} - \bar{u}\|$$

$$\|\bar{u} - \bar{v}\|$$

$$A = \begin{bmatrix} \bar{u}_1 & \dots & \bar{u}_m \end{bmatrix}$$

$\mathrm{Col}\,A = \mathrm{span}\{\bar{u}_1, \dots, \bar{u}_m\}$ er mængden af vektorer på formen

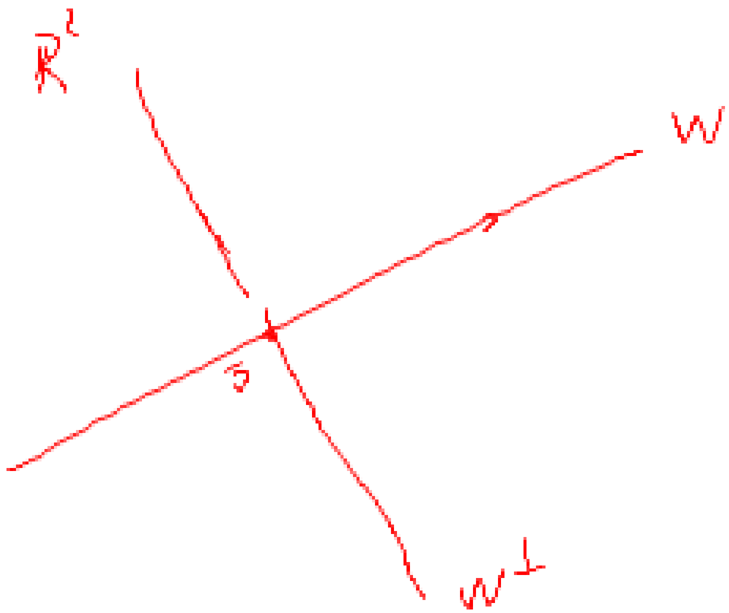
$$x_1\bar{u}_1 + x_2\bar{u}_2 + \dots + x_m\bar{u}_m =$$

$$\begin{bmatrix} \vdots & \dots & \bar{u}_m \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = A\bar{x}$$

$A\bar{x} = \bar{b}$ har løsning hvis $\bar{b} \in \mathrm{Col}\,A$

Ellers find $\hat{b}$ i $\mathrm{Col}\,A$ tættest på $\bar{b}$

og løs $A\bar{x} = \hat{b}$



$$(W^\perp)^\perp = W$$

$W = \mathbb{R}^n \qquad W^\perp = \{\bar{0}\}$

$W = \{\bar{0}\} \qquad W^\perp = \mathbb{R}^n$

$$A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 2 & 3 & -2 & 1 \\ 3 & 1 & -10 & 12 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & -1 & -2 & 3 \\ 0 & -5 & -10 & 15 \end{bmatrix} \sim$$

$$\begin{bmatrix} \underline{1} & 0 & -4 & 5 \\ 0 & \underline{1} & 2 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Col A basis: $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} \right\}$

Row A has basis: $\left\{ \begin{bmatrix} 1 \\ 0 \\ -4 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ -3 \end{bmatrix} \right\}$

Nul A

free variables. $x_4 = s$
$x_3 = t$

$x_2 + 2t - 3s = 0$

$x_1 - 4t + 5s = 0$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = s \begin{bmatrix} -5 \\ 3 \\ 0 \\ 1 \end{bmatrix} + t \begin{bmatrix} 4 \\ -2 \\ 1 \\ 0 \end{bmatrix}$$

$$(\text{Row } A)^{\perp} = \text{Nul } A$$

$\bar{x} \in \text{Nul } A$

$$\begin{bmatrix} - & \bar{a}_1 & - \\ & \vdots & \\ - & \bar{a}_m & - \end{bmatrix} \bar{x} = \bar{0} \iff \bar{a}_1 \cdot \bar{x} = 0, \ldots, \bar{a}_m \cdot \bar{x} = 0 \iff$$

$\bar{x}$ ortogonal på radene i A

$\Updownarrow$

$\bar{x}$ ortogonal på Row A

$\bar{x} \in (\text{Row } A)^{\perp}$

$$\bar{y} - \bar{z} = \hat{y}$$

$$\hat{y} = c_1 \bar{u}_1 + \dots + c_p \bar{u}_p$$

$$\bar{u}_1 \cdot \hat{y} = \bar{u}_1 \cdot (c_1 \bar{u}_1 + \dots + c_p \bar{u}_p)$$

$$= c_1 \bar{u}_1 \cdot \bar{u}_1 + 0 + \dots + 0$$

$$= c_1 \bar{u}_1 \cdot \bar{u}_1$$

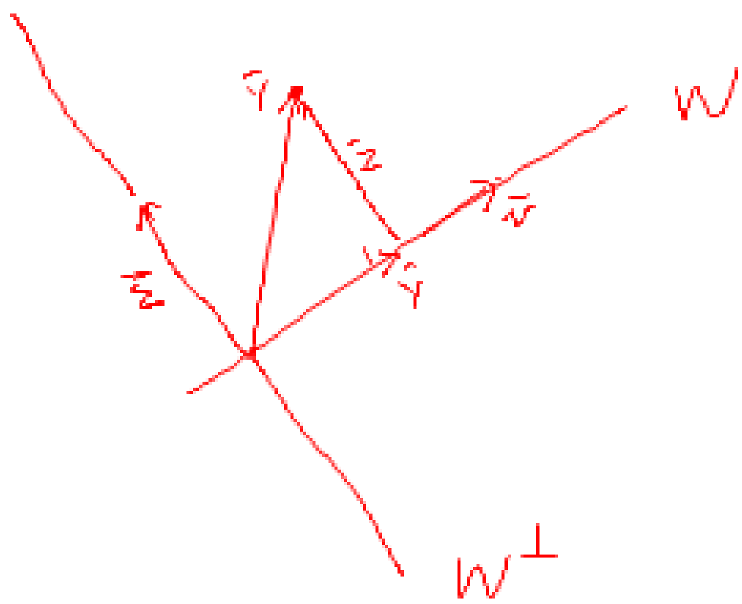
$$\bar{z} \in W^{\perp} \qquad \bar{u}_1 \cdot \bar{z} = 0$$

$$\bar{u}_1 \cdot \hat{y} = \bar{u}_1 \cdot (\bar{y} - \bar{z}) = \bar{u}_1 \cdot \bar{y} - \bar{u}_1 \cdot \bar{z}$$

$$= \bar{u}_1 \cdot \bar{y}$$

$$c_1 \bar{u}_1 \cdot \bar{u}_1 = \bar{u}_1 \cdot \bar{y}$$

$$c_1 = \frac{\bar{u}_1 \cdot \bar{y}}{\bar{u}_1 \cdot \bar{u}_1}$$



$\vec{y}$
$\vec{z}$
$\vec{w}$
$\hat{y}$
$\vec{u}$
W
$W^\perp$