MCG - 4

The polar coordinates for a point (x, y) in the plane is (r, θ) where $r = \sqrt{x^2 + y^2}$ is the distance from $(0, 0)$ to (x, y), and θ is the angel (in positive direction) from the x-axis to the vector (x, y).

Converting from (r, θ) to (x, y):

$$x = r \cos\theta, \qquad y = r \sin\theta.$$

Converting from (x, y) to (r, θ):

$$r = \sqrt{x^2 + y^2}, \qquad \theta = \begin{cases} \arctan\frac{y}{x} & \text{hvis } x > 0, \\ \arctan\frac{y}{x} + \pi & \text{hvis } x < 0, \\ \frac{\pi}{2} & \text{hvis } x = 0, y > 0, \\ -\frac{\pi}{2} & \text{hvis } x = 0, y < 0. \end{cases}$$

If you prefer to work with degrees then replace π by 180°.

The spherical coordinates for a point $P = (x, y, z)$ in space are (ρ, ϕ, θ) where $\rho = \sqrt{x^2 + y^2 + z^2}$ is the distance from $(0, 0, 0)$ to (x, y, z), and ϕ is the angel between the z-axis and the vector (x, y, z).
$0 \le \phi \le \pi$ (or $0 \le \phi \le 180^\circ$). θ is the same as in polar coordinates for (x, y).

Converting from (ρ, ϕ, θ) to (x, y, z):

$$x = \rho \sin\phi \cos\theta, \quad y = \rho \sin\phi \sin\theta, \quad z = \rho \cos\phi.$$

Converting from (x, y, z) to (ρ, ϕ, θ):

$$\rho = \sqrt{x^2 + y^2 + z^2}, \quad \phi = \arccos \frac{z}{\rho}.$$

θ is computed as on the previous page.

A line passing through points P_0 and P_1 consisits of points that can be written in parametric form as

$$P_0 + t\mathbf{d}, \quad t \in \mathbb{R},$$

where $\mathbf{d} = P_1 - P_0$ is the vector from P_0 to P_1.

For a line in the plane there a vector $\mathbf{n} = (a, b)$ (e.g. if $\mathbf{d} = (b, -a)$) perpendicular to the line.

A point $Q = (x, y)$ lies on the line if and only if

$$\mathbf{n} \cdot (Q - P_0) = 0.$$

If $P_0 = (x_0, y_0)$ then this equation can be written as

$$ax + by + c = 0,$$

where $c = -ax_0 - by_0$. This is called a generalized line equation.

If $||\mathbf{n}|| = \sqrt{a^2 + b^2} = 1$ and $ax + by + c = d$ then the point (x, y) is in distance $|d|$ from the line – if $d > 0$ on the same side of the line as indicated by $\mathbf{n}$.

A plane passing through the points P_0, P_1, P_2 consists of points that can be written in parametric form as

$$P_0 + s\mathbf{u} + t\mathbf{v}, \quad s, t \in \mathbb{R},$$

where $\mathbf{u} = P_1 - P_0$ and $\mathbf{v} = P_2 - P_0$.

For a plane in $\mathbb{R}^3$ there is a vector $\mathbf{n} = (a, b, c)$ (e.g. $\mathbf{n} = \mathbf{u} \times \mathbf{v}$) perpendicular to the plane.

A point $Q = (x, y, z)$ lies on the plane if and only if

$$\mathbf{n} \cdot (Q - P_0) = 0.$$

If $P_0 = (x_0, y_0, z_0)$ then this equation can be written as

$$ax + by + cz + d = 0,$$

where $d = -ax_0 - by_0 - cz_0$. This is called a generalized plane equation.

If $||\mathbf{n}|| = \sqrt{a^2 + b^2 + c^2} = 1$ and (x, y, z) is an arbitrary point in space then $|ax + by + cz + d|$ is the distance between the point and the plane – if $ax + by + cz + d > 0$ on the same side of the plane as indicated by $\mathbf{n}$.

Let P be a point on the plane passing through P_0, P_1, P_2.
Then there exists unique numbers s, t so that

$$P = P_0 + s\mathbf{u} + t\mathbf{v}, \quad \text{where } \mathbf{u} = P_1 - P_0 \text{ and } \mathbf{v} = P_2 - P_0.$$

If $\mathbf{w} = P - P_0 = s\mathbf{u} + t\mathbf{v}$ then s and t can be determined from the equations

$$\mathbf{v} \times \mathbf{w} = s(\mathbf{v} \times \mathbf{u}), \quad \mathbf{u} \times \mathbf{w} = t(\mathbf{u} \times \mathbf{v}).$$

Then

$$P = P_0 + s(P_1 - P_0) + t(P_2 - P_0) = (1 - s - t)P_0 + sP_1 + tP_2.$$

Thus the barycentric coordinates for P are $(1 - s - t, s, t)$.

If P is inside the triangle with vertices P_0, P_1, P_2 then $1 - s - t \geq 0, s \geq 0, t \geq 0$.
If one of the numbers is negative then P is outside the triangle.