MCG - 13

Converting from rotation matrix to normalized quaternion:

R: a 3×3 rotation matrix.

Compute:

$$q = (R_{00} + R_{11} + R_{22} + 1, R_{21} - R_{12}, R_{02} - R_{20}, R_{10} - R_{01}).$$

The rotation is then represented by the normalized quaternion

$$\frac{1}{||q||} q.$$

Alternative method (if trace(R) < 0):
Find the largest of the numbers R_{00}, R_{11}, R_{22}.

R_{00} largest: normalize the quaternionen

$$(R_{21} - R_{12}, R_{00} - R_{11} - R_{22} + 1, R_{01} + R_{10}, R_{02} + R_{20}).$$

R_{11} largest: normalize the quaternionen

$$(R_{02} - R_{20}, R_{01} + R_{10}, R_{11} - R_{00} - R_{22} + 1, R_{12} + R_{21}).$$

R_{22} largest: normalize the quaternionen

$$(R_{10} - R_{01}, R_{02} + R_{20}, R_{21} + R_{12}, R_{22} - R_{00} - R_{11} + 1).$$

If rotation around the axis $\mathbf{r}_1$ with angle θ_1 is represented by the quaternion q_1
and rotation around the axis $\mathbf{r}_2$ with angle θ_2 is represented by the quaternion q_2

then the composed rotation consisting of
    rotation around the axis $\mathbf{r}_1$ with angle θ_1
followed by
    rotation around the axis $\mathbf{r}_2$ with angle θ_2
is represented by the quaternion $q_2 q_1$.

Linear interpolation:

Find a parameterized line $Q(t)$, satisfying that $Q(t_i) = P_i$ and $Q(t_{i+1}) = P_{i+1}$, where P_i and P_{i+1} are points.

Solution

$$Q(t) = P_i + \frac{t - t_i}{t_{i+1} - t_i}(P_{i+1} - P_i),$$

when $t_i \leq t \leq t_{i+1}$.

Hermite curves:
Determine a curve $Q(t)$ satisfying that $Q(0) = P_0$, $Q(1) = P_1$, $\mathbf{Q}'(0) = \mathbf{P}'_0$ and $\mathbf{Q}'(1) = \mathbf{P}'_1$, where P_0 and P_1 are points and $\mathbf{P}'_0$ and $\mathbf{P}'_1$ are vectors.

Let $Q(t) = \mathbf{a}t^3 + \mathbf{b}t^2 + \mathbf{c}t + D$, where $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are vectors and D is a point.
Then $\mathbf{Q}'(t) = 3\mathbf{a}t^2 + 2\mathbf{b}t + \mathbf{c}$.

Requirement:

$$Q(0) = D = P_0, \quad Q(1) = \mathbf{a} + \mathbf{b} + \mathbf{c} + D = P_1.$$
$$\mathbf{Q}'(0) = \mathbf{c} = \mathbf{P}'_0, \quad \mathbf{Q}'(1) = 3\mathbf{a} + 2\mathbf{b} + \mathbf{c} = \mathbf{P}'_1.$$

Solution:
$\mathbf{a} = 2(P_0 - P_1) + \mathbf{P}'_0 + \mathbf{P}'_1$, $\mathbf{b} = 3(P_1 - P_0) - 2\mathbf{P}'_0 - \mathbf{P}'_1$,
$\mathbf{c} = \mathbf{P}'_0$ and $D = P_0$.

The Hermite curve satisfying the above condition can also be written as

$$Q(u) = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ \mathbf{P}'_0 \\ \mathbf{P}'_1 \end{bmatrix} = UMG,$$

where the 'vector' G is in fact a 4×3 matrix.