Exam - Mathematics for Computer Graphics

Medialogy, 5. semester, Aalborg University

Thursday, January 5, 2012, 9.00–13.00.

You are allowed to use: books, notes and a calculator during the exam.
You are *not* allowed to use: computers or phones.

A reasoned explanation should follow the solution of the exercises. Moreover, the intermediate steps leading to the solution should also be written down. You can write the exam in Danish or English.
The percentage following each exercise number stands for the exercises's value in the final mark.

Exercise 1 (15 %)

A rotation R appears by concatenating three axis rotations: first we rotate by $90°$ around the z-axis, then we rotate by $180°$ around the y-axis, and finally we rotate by $90°$ around the x-axis.

1. Determine the matrix representing the rotation R.
2. To which new vector is $\mathbf{v} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$ rotated by the rotation R.

Exercise 2 (8 %)

Let $\mathbf{v} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$ and $S : \mathbb{R}^3 \mapsto \mathbb{R}^3$ be the linear transformation $S(\mathbf{x}) = \mathbf{v} \times \mathbf{x}$.
Find a matrix A so that $S(\mathbf{x}) = A\mathbf{x}$ for all $\mathbf{x} \in \mathbb{R}^3$.

Exercise 3 (20 %)

Consider the following points: $P_0 = (-1, 0, 1)$, $P_1 = (-1, 2, 2)$, and $P_2 = (4, 1, -1)$.

1. Determine an equation for the plane through the points P_0, P_1, P_2.
2. Show that $P = (0, 1, 1)$ is a point on the plane.
3. Determine the barycentric coordinates for P (with respect to P_0, P_1, P_2).

Exercise 4 (16 %)

In this exercise vi consider the Hermite curve $Q(u)$, $u = 0, \ldots, 1$, satisfying that

$$Q(0) = P_0 = (1,0,0)^T$$
$$Q(1) = P_1 = (2,1,2)^T$$
$$Q'(0) = \mathbf{P}'_0 = (3,-2,1)^T$$
$$Q'(1) = \mathbf{P}'_1 = (-1,2,1)^T.$$

1. Determine the curve $Q(u)$.
2. Compute $Q(\frac{1}{2})$.

Exercise 5 (11 %)

Consider the quaternions $p = (0, \frac{3}{5}, 0, -\frac{4}{5})$ and $q = (\frac{4}{5}, 0, \frac{3}{5}, 0)$.

- Determine the function $\mathrm{slerp}(p, q, t)$.
- Compute $\mathrm{slerp}(p, q, \frac{2}{3})$.

Exercise 6 (5 %)

Determine the Cartesian coordinates (x, y, z) for the point with spherical coordinates $(\rho, \phi, \theta) = (3, \frac{\pi}{4}, \frac{\pi}{2})$, where the angles are given in radians.

Exercise 7 (7 %)

Determine the quaternion representing the rotation with axis $\mathbf{r} = \begin{bmatrix} 3 \\ -6 \\ 2 \end{bmatrix}$ and angle $\theta = 60°$.

Exercise 8 (18 %)

Consider the matrices $S = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $R = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$.

1. Compute SS^T and RR^T. Show that S and R are both orthogonal matrices.

2. Compute the determinant of S and R. Which one of the matrices S and R is a rotation matrix.

3. Determine rotation axis and angle for that rotation matrix.