Session 13, April 4, 2011, 12:30–16:15

Program

1. 12:30–14:00 in G5-112. We now start on the core topic of the course, the theorems named after Cauchy (Augustin-Louis Cauchy, 1789-1857). I will go through most of section 5 in [AJ].

2. 14:00–16:15 in groups. See the list of exercises below.

Exercises

1. From [AJ] section 4.1 Exercises 1, 2, 3, 4.

2. The exercises on the list below.

3. Any exercises left from sessions 11 and 12.

Review of complex numbers. It may be useful for some of you to review the material on complex numbers and polynomials from the first year calculus course. The texts used were

- E.B. Saff et al. Complex numbers and differential equations, Custom print (2nd edition), Pearson, 2009.

- A. Jensen. Lecture notes on polynomials. Second edition 2009 (9 pages). It is available here.

We will soon use all the results on polynomials with complex coefficients given in the second text.

Exercises

1. Let $\gamma\colon [a,b] \to \mathbf{C}$ be a circuit. Define τ by

$$\tau(t) = \gamma(b+a-t), \quad t \in [a,b].$$

Show that $\tau^* = \gamma^*$. Explain why τ is a circuit.

Let $f\colon \gamma^* \to \mathbf{C}$ be a continuous function. Show that

$$\int_\tau f(z)dz = \int_\gamma f(z)dz.$$

We usually denote τ by $-\gamma$. It is the path γ traversed in the opposite direction.

2. Let $\gamma_1\colon [a,b] \to \mathbf{C}$ and $\gamma_2\colon [c,d] \to \mathbf{C}$ be two circuits. Assume that $\gamma_1(b) = \gamma_2(c)$. Define

$$\tau(t) = \begin{cases} \gamma_1(t), & t \in [a,b], \\ \gamma_2(t+c-b), & t \in [b, d-c+b]. \end{cases}$$

Try to explain why τ is a circuit. Show that $\tau^* = \gamma_1^* \cup \gamma_2^*$.

Let $f\colon \gamma_1^* \cup \gamma_2^* \to \mathbf{C}$ be a continuous function. Show that

$$\int_\tau f(z)dz = \int_{\gamma_1} f(z)dz + \int_{\gamma_2} f(z)dz.$$

The circuit τ is called the concatenation of the two circuits γ_1 and γ_2. It is usually denoted by $\gamma_1 \cup \gamma_2$.

As an example, take

$$\gamma_1(t) = 2e^{it}, \quad t \in [0, \tfrac{\pi}{2}],$$
$$\gamma_2(t) = -(2+2i)t + 2i, \quad t \in [0, 1].$$

Sketch the two circuits in the complex plane and compare with τ defined by the method given above.

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