Session 14, April 7, 2011, 12:30–16:15

Program

1. 12:30–14:00 in G5-112. I will first review the main results from section 5, and then go through section 6 on applications of the Cauchy integral formula in [AJ].

2. 14:00–16:15 in groups. See the list of exercises below.

Exercises

1. From [AJ] section 5.1 Exercises 1, 2, 3.

2. Carry out the details in Example 5.8 in [AJ].

3. Solve the following problems

    (a) Let $f(z) = z^3 - z - 1$ and let γ be any circuit in the complex plane connecting the points -1 and $2i$. Find $\int_\gamma f(z)dz$.

    (b) Let $g(z) = (z^2 + 1)^{-1}$. Let γ_1 be the circle with center 0 and radius $\frac{1}{2}$ traversed once with positive orientation. Find $\int_{\gamma_1} g(z)dz$.

    Let γ_2 be the circle with center $-i$ and radius $\frac{1}{2}$ traversed once with positive orientation. Find $\int_{\gamma_2} g(z)dz$.

    Let γ_3 be the circle with center -4 and radius 2 traversed once with positive orientation. Find $\int_{\gamma_3} g(z)dz$.

Review exercises: *Revised April 4.* Below is a collection of exercises that review essential techniques for finding roots in polynomials and factoring polynomials. It is **strongly recommended** to do these exercises! If you have not yet done so, read again the lecture notes

   A. Jensen. Lecture notes on polynomials. Second edition 2009 (9 pages). They are available here.

   For each of the polynomials below find all its roots and write it as a product of polynomials of degree 1.

1. $z^2 - 1$

2. $z^2 + 16$

3. $z^2 + 2z + 2$

4. $z^2 + z + 1 - i$ Note: Polynomial changed.

5. $z^3 - 1$

6. $z^3 + 27$

7. $z^4 - 16$

8. $z^4 + 1$

9. $z^4 - z^2 - 2$

Solutions to review exercises   Below are the factorizations of the polynomials in the review exercises.

1. $(z-1)(z+1)$

2. $(z+4i)(z-4i)$

3. $(z-1-i)(z-1+i)$

4. $(z-i)(z+1+i)$

5. $(z-1)(z+\frac{1}{2}+i\frac{1}{2}\sqrt{3})(z+\frac{1}{2}-i\frac{1}{2}\sqrt{3})$

6. $(z+3)(z-\frac{3}{2}+i\frac{3}{2}\sqrt{3})(z-\frac{3}{2}-i\frac{3}{2}\sqrt{3})$

7. $(z-2)(z+2)(z-2i)(z+2i)$

8. $(z-\frac{1}{2}\sqrt{2}-i\frac{1}{2}\sqrt{2})(z-\frac{1}{2}\sqrt{2}+i\frac{1}{2}\sqrt{2})(z+\frac{1}{2}\sqrt{2}-i\frac{1}{2}\sqrt{2})(z+\frac{1}{2}\sqrt{2}+i\frac{1}{2}\sqrt{2})$

9. $(z-\sqrt{2})(z+\sqrt{2})(z-i)(z+i)$

<div align="right">Arne Jensen</div>