Session 16, April 12, 2011, 12:30–16:15

Program

1. 12:30–14:00 in G5-112. I will introduce the concept of the residue at a pole of a meromorphic function and then prove the residue theorem, section 8 in [AJ].
2. 14:00–16:15 in groups. See the list of exercises below.

Exercises

1. For the functions $\exp(z)$, $\cos(z)$, and $\sin(z)$, you should explain why they are entire functions and then find their power series expansions around zero. In these particular cases you can use the Taylor expansion formula (6.1) from [AJ]. Note that you will need these power series expansions in solving other problems, *including exam problems.*
2. From [AJ] section 7.1 Exercise 2.
3. Find the radius of convergence of the power series expansion of $f(z) = \dfrac{z}{z^2+4}$ around each of the points 0, $1+i$ and $4-2i$. Note: You do not need to *find the expansions* to find the radii of convergence.
4. Find the domain of definition of the function $g(z) = \tan(z)$, explain why it is holomorphic on this domain, and then find the radius of convergence of the power series expansion of $g(z)$ around each of the points 0, $\pi/4$, and i. Note: You do not need to *find the expansions* to find the radii of convergence.
5. Find the power series expansion of the function $h(z) = \dfrac{1}{16+z^4}$ around the point 0. What is its radius of convergence?
6. Find the power series expansion of the function $u(z) = \dfrac{z}{(1+z^2)^2}$ around the point 0. *Hint:* Start by finding a primitive of the function $u(z)$. What is its radius of convergence?

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