Session 2, February 7, 2011, 12:30–16.15 Note that this semester we put a lot of emphasis on *solving problems*. Thus you are required to solve all the exercises posed for each session. It is a good idea to take the extra time to write down a complete solution to each exercise, for future reference.

Program

1. 12:30–14:00 in G5-112. Lecture on sections 9.4 and 9.5 in [PF]. I will use some results from integration theory. They will be covered in detail later in the course. Thus parts of the proofs will be postponed to later sessions. I will not go through Proposition 9.35.

2. 14:00–15:30 in groups. See the list of exercises below.

3. 15:30-16:15 in G5-112. Review of solutions to exercises today. Note that the concept of uniform convergence is difficult. I will review some of the techniques used in solving exercises involving uniform convergence.

Exercises Solve the exercises in the order posed.

1. Section 9.3, Exercises 1, 2, 3, 4.

2. Section 9.3, Exercise 5. Before this session you should read carefully the three examples covered in this exercise.

3. Exercise A, see below.

4. See the list below with exercises on infinite series.

Exercise A The formulation of the Weierstrass Uniform Convergence Criterion in [PF] is not convenient to use in solving problems involving infinite series of functions.

Prove the following theorem.

Theorem. *Let D be a set, and let $f_k\colon D \to \mathbf{R}$, $k \geq 0$, be a sequence of functions. Assume there exists constants $M_k \geq 0$ and $k_0 \in \mathbf{N}$, such that*

$$|f_k(x)| \leq M_k \quad \text{for all } k \geq k_0 \text{ and all } x \in D. \tag{1}$$

Assume that $\sum_{k=k_0}^{\infty} M_k$ is convergent. Then the series $\sum_{k=0}^{\infty} f_k(x)$ is absolutely and uniformly convergent on D.

This theorem is often called the *Weierstrass M-test.* We will use the result is this form many times in the future.

Exercises on infinite series. For each of the series below, determine whether it is convergent, absolutely convergent, or divergent. Furthermore, for the exercises 1., 3., and 4., find the sum of the series in each case.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{3^{n+1}}$.

2. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n!}$.

3. $\sum_{n=1}^{\infty}(\frac{1}{n}-\frac{1}{n+1})$.

4. $\sum_{n=1}^{\infty}(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}})$.

5. $\sum_{n=2}^{\infty}\frac{1}{n\ln n}$.

6. $\sum_{n=2}^{\infty}\frac{(-1)^n}{n\ln n}$.

7. $\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^2}$.

8. $\sum_{n=2}^{\infty}\frac{(-1)^n}{n(\ln n)^2}$.

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