Session 6, March 3, 2011, 12:45–16:30

Program

1. 12:45–14:15 in G5-112. Today I prove the existence and uniqueness theorem for differential equations, [PF], section 12.3. I will also review some results concerning complete metric spaces and the contraction mapping theorem, [PF], section 12.2. The result in [PF] is formulated for a single equation, but we will need the result for a system of differential equation. The result is formulated below.

2. 14:15–16:30 in groups. See the list of exercises below. Note that there is extra time for solving problems today.

Exercises Solve the exercises in the order posed.

1. Section 6.5, Exercises 2 and 5.
2. Section 6.6, Exercises 1 and 3.
3. Section 7.1, Exercises 1 and 2.
4. Exam June 2009, Opgave 1.
5. Trial Exam June 2009, Opgave 1.

Important! Write down complete solutions to the two exam problems posed today. I will check the written solutions while visiting the groups, either today, or next session.

Vector valued differential equation. The vector valued version of the existence and uniqueness theorem:

Assumption. *Let $I \subseteq \mathbf{R}$ be a non-empty open interval, let $\Omega \subseteq \mathbf{R}^n$ be a non-empty open set, and let $\mathbf{F}\colon I \times \Omega \to \mathbf{R}^n$ be a continuous function, which for $M > 0$ satisfies the Lipschitz condition*

$$\|\mathbf{F}(x, \mathbf{y}_1) - \mathbf{F}(x, \mathbf{y}_2)\| \leq M\|\mathbf{y}_1 - \mathbf{y}_2\| \tag{1}$$

for all $x \in I$ and all $\mathbf{y}_1, \mathbf{y}_2 \in \Omega$.

The system of n first order differential equations is written in vector valued form as

$$\frac{d\mathbf{y}}{dx}(x) = \mathbf{F}(x, \mathbf{y}), \tag{2}$$

$$\mathbf{y}(x_0) = \mathbf{y}_0. \tag{3}$$

Here we assume $\mathbf{y}_0 \in \Omega$ and $x_0 \in I$.

Theorem. *Let $\mathbf{F}$ satisfy the above Assumption. For each $(x_0, \mathbf{y}_0) \in I \times \Omega$ there exists a $\delta > 0$, such that (2) with the initial condition (3) has a unique continuously differentiable solution, defined in the interval $(x_0 - \delta, x_0 + \delta)$.*

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